Exact Query Reformulation with First-order Ontologies and Databases

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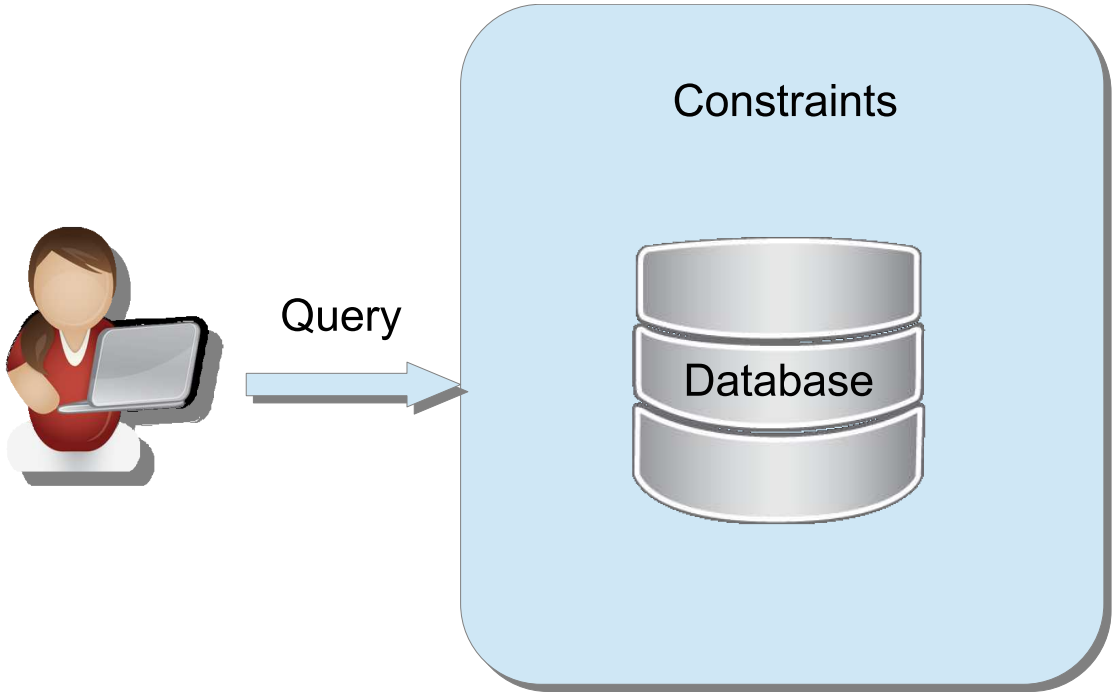
A joint work with Enrico Franconi and Volga Kernet

Nov 13, 2013

Query Answering Under Contraints and Databases



- Given a FOL fragment $\mathcal{L}$
- Constraints: a set of L-sentences $\mathcal{KB}$
- Database: a finite set of facts $\mathcal{DB}$ over a relational signature $\mathcal{P}_{\mathcal{DB}}$
- Query: a (open) $\mathcal{L}$-formula

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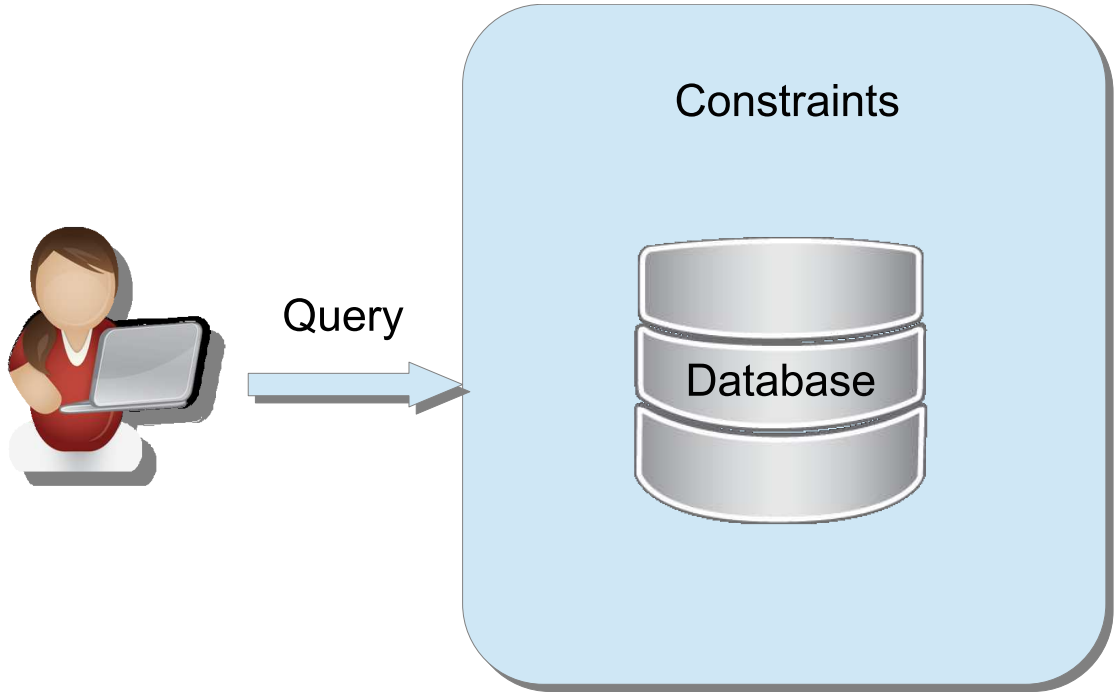

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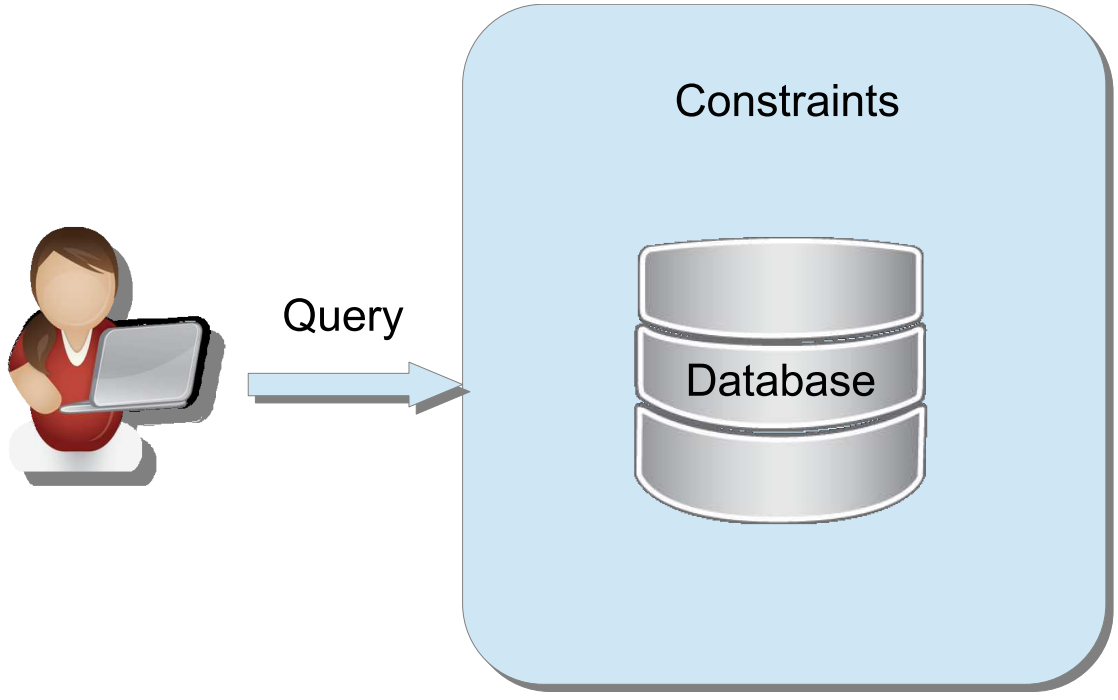

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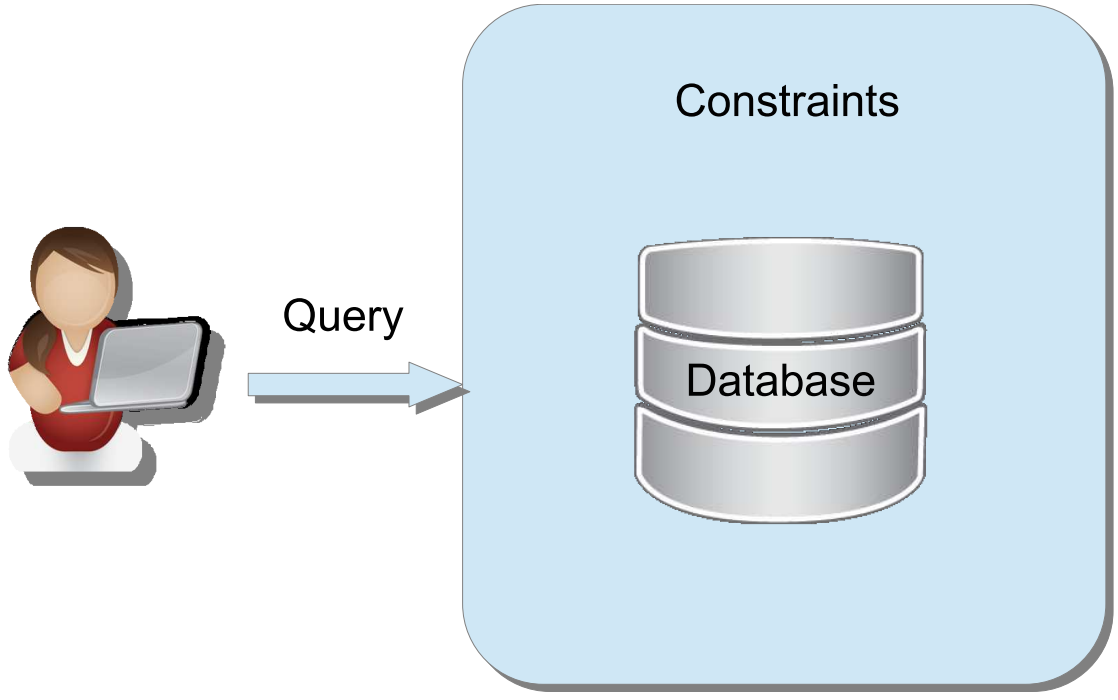

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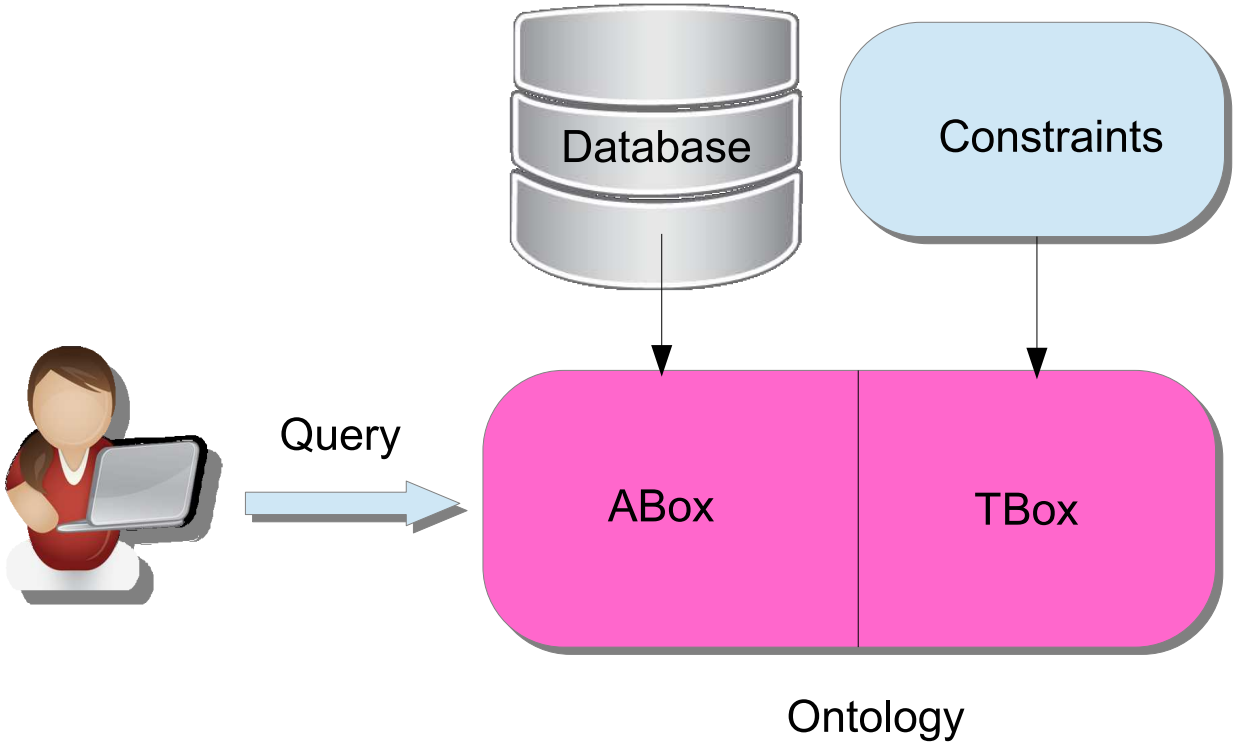

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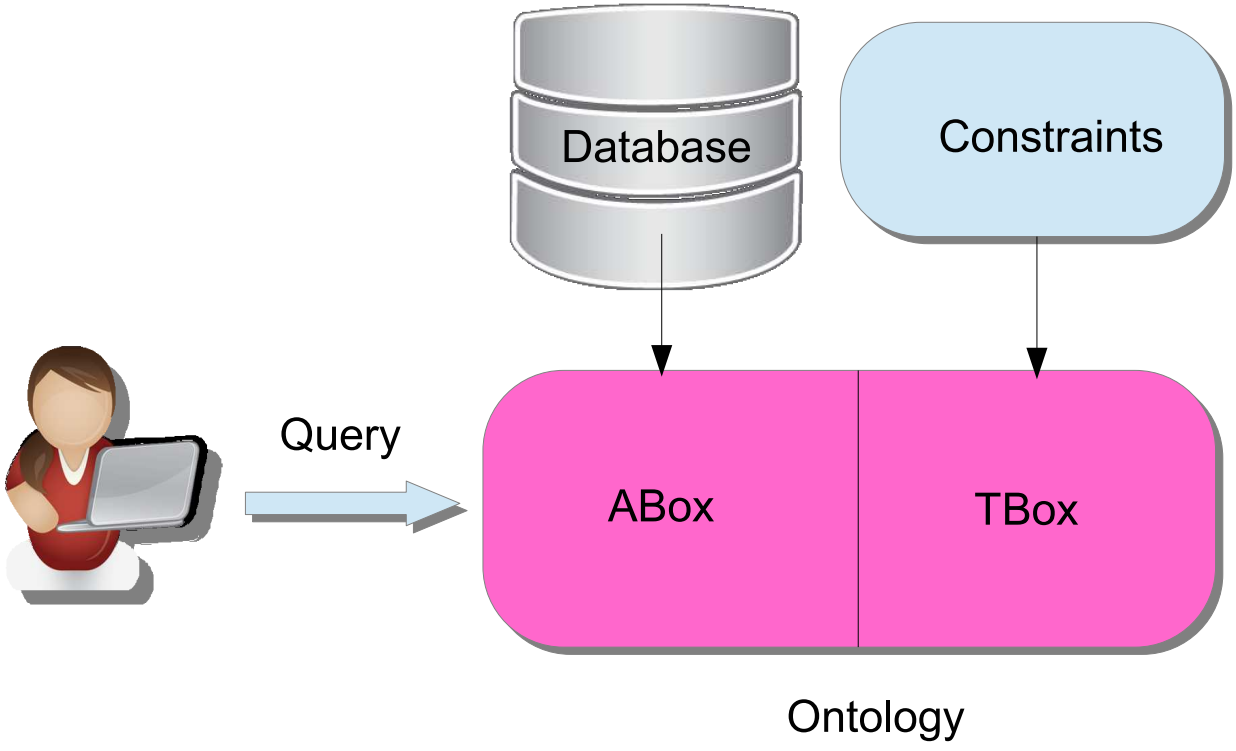

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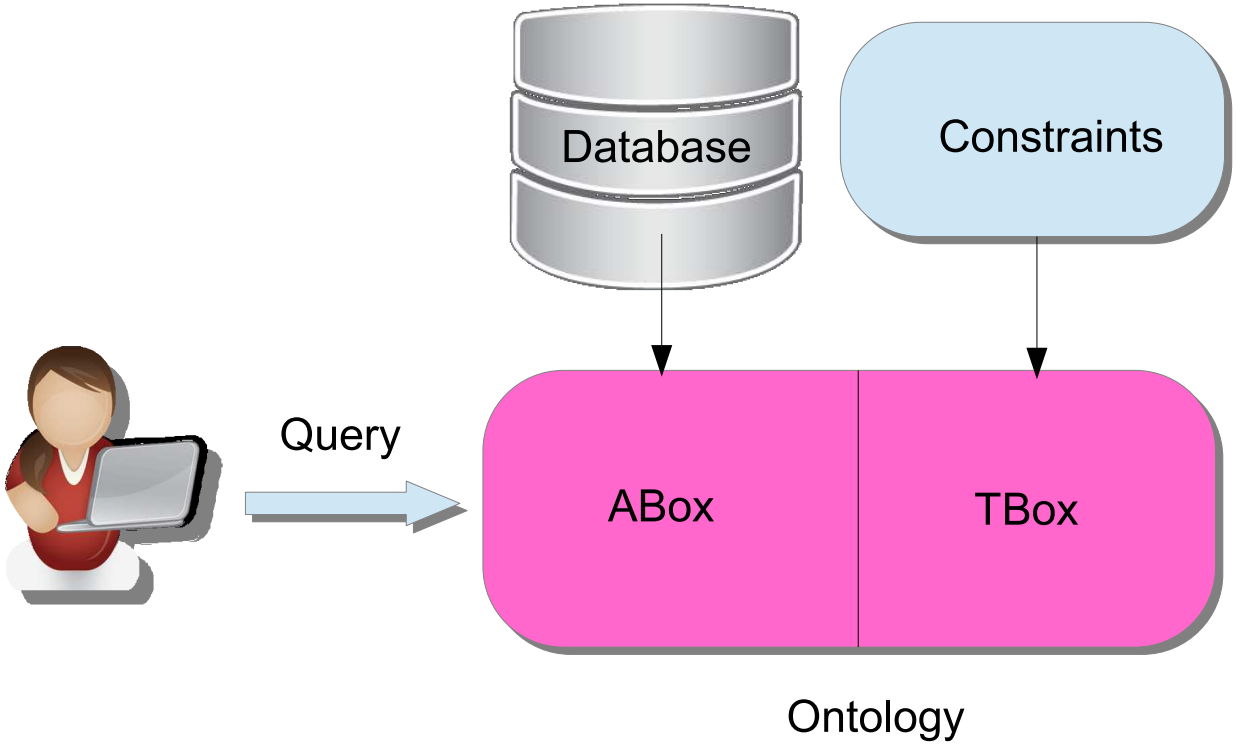

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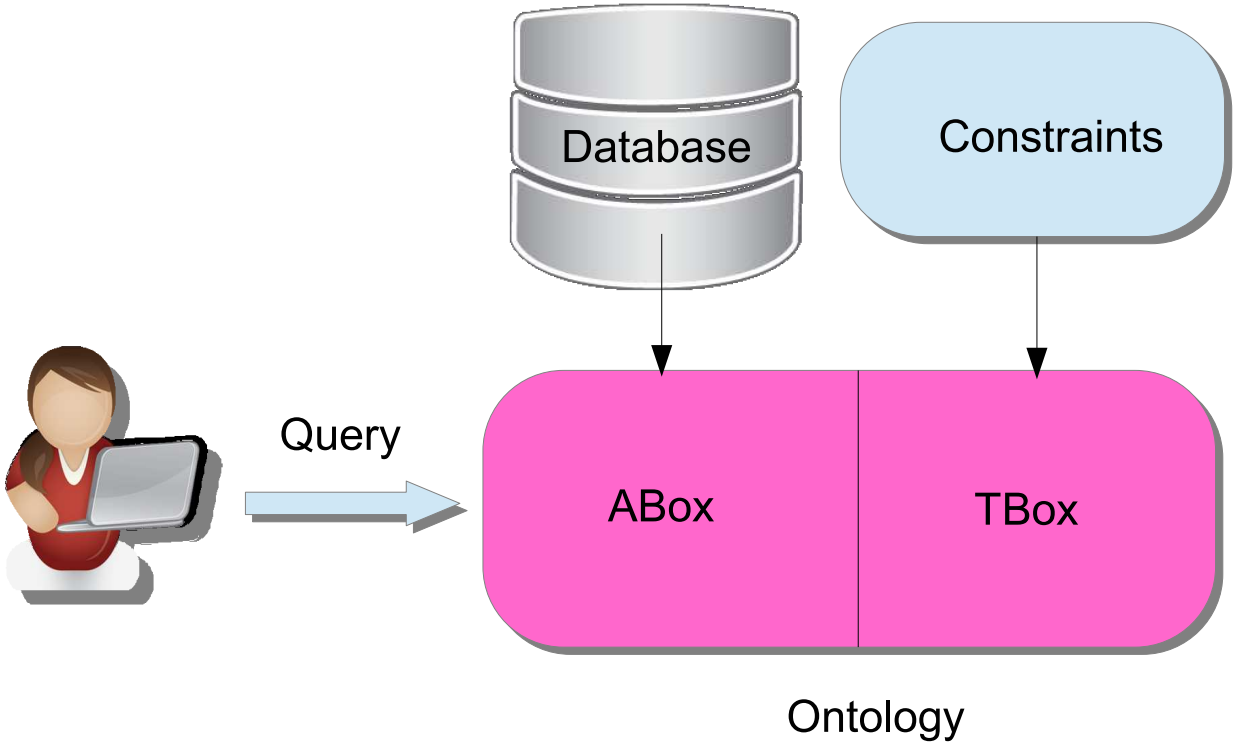

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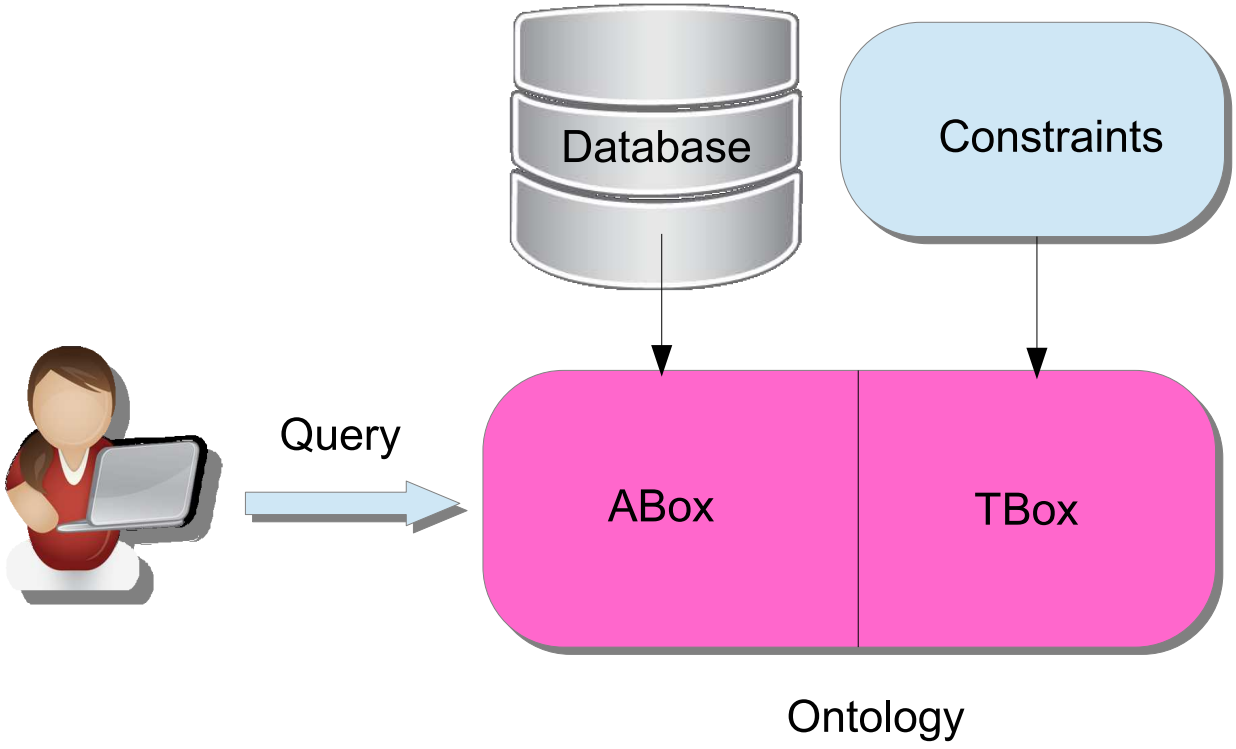

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ABox

- An ABox is a finite set of ground atomic facts, syntactically

Example

$\mathcal{A} = \{\texttt{Employee(Jon)}, \texttt{Project(Winter)}\}$ is an ABox

- The semantics of ABoxes is given by first-order structures.
- An ABox does not correspond to a first-order structure!

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ABox Semantics

Definition

A first-order structure $\mathcal{I}$ is a model of an ABox $\mathcal{A}$ if $\mathcal{I} \supseteq \mathcal{A}$

An ABox is an incomplete database i.e., an ABox represents a class of databases.

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Given the ABox $\mathcal{A} = \{\texttt{Employee(Jon)}, \texttt{Project(Winter)}\}$

- $\{\{\texttt{Employee(Jon)}, \texttt{Project(Winter)}\}$ is a model of $\mathcal{A}$
- $\{\{\texttt{Employee(Jon)}, \texttt{Employee(Rob)}, \texttt{Project(Winter)}\}$
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Certain Answers

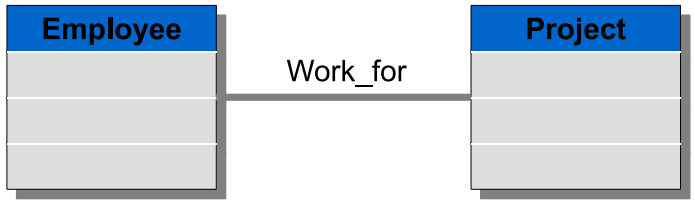
Definition

Given:

- a TBox $\mathcal{T}$, an ABox $\mathcal{A}$ and
- a concept query $\mathcal{Q}$ and an individual a

Question: $(\mathcal{T}, \mathcal{A}) \models \mathcal{Q}(a)$? That is $\forall$ models $\mathcal{I}$ of $(\mathcal{T}, \mathcal{A})$, is it the case that $\mathcal{I} \models \mathcal{Q}(a)$

ABox vs Database



- Additional constraint as a standard view over data:
  $\forall x$`Bad-Project(x)` $\leftrightarrow$ `Project(x)` $\wedge \neg \exists y$`Work-for(y,x)`
- Database:
  `Work-for` $= \{$`<Jon, Winter>, <Rob, Winter>`$\}$
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ABox does not scale down to standard DB answer

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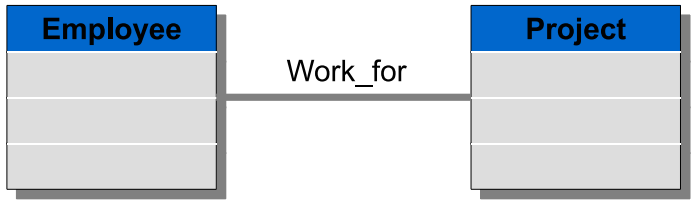

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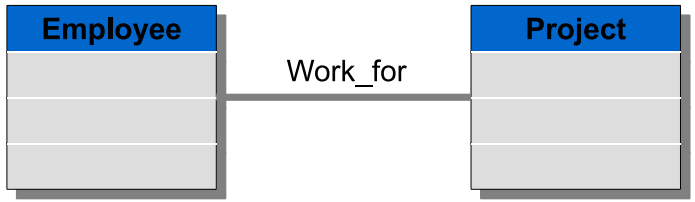

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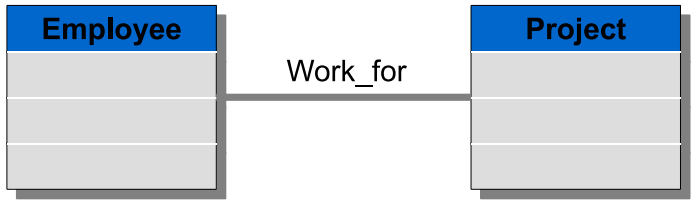

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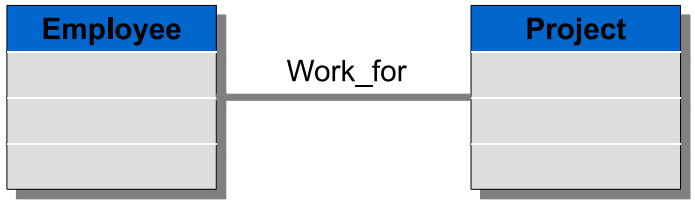

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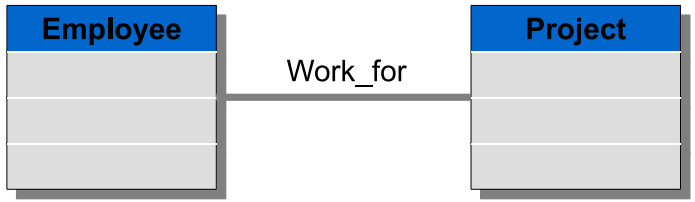

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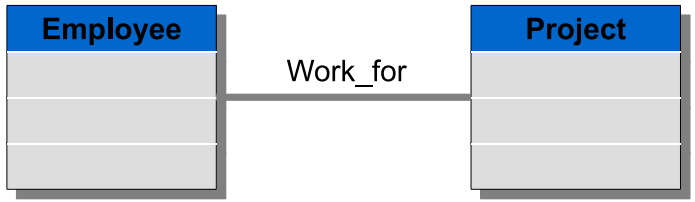

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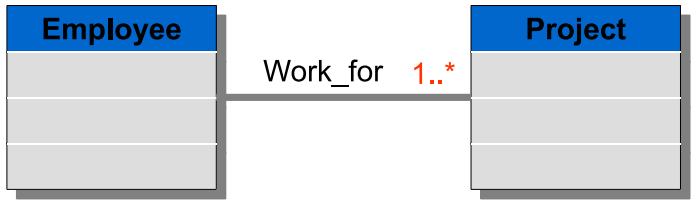

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- Query as a standard view over database:

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  - $Q =\ \texttt{EVAL}(\Pi_2\ \texttt{Work-for})$

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Queries are not compositional wrt certain answer semantics!

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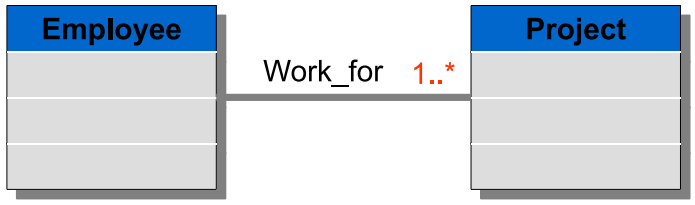

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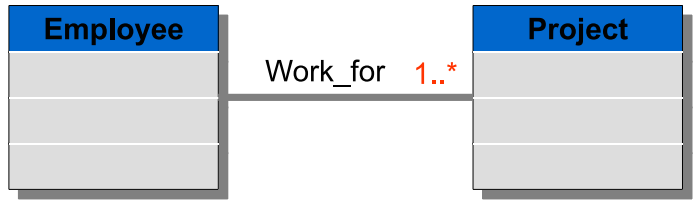

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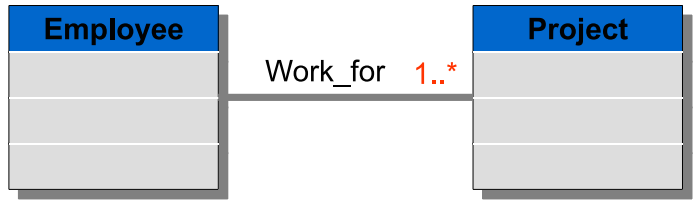

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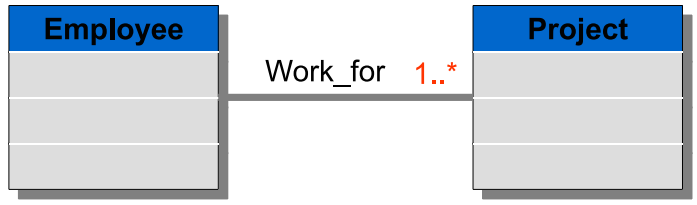

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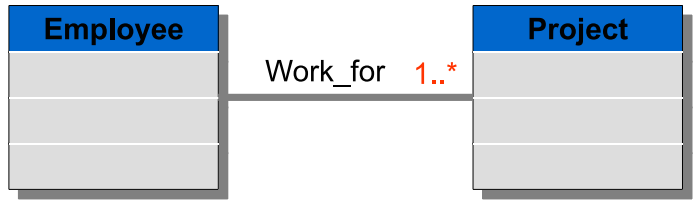

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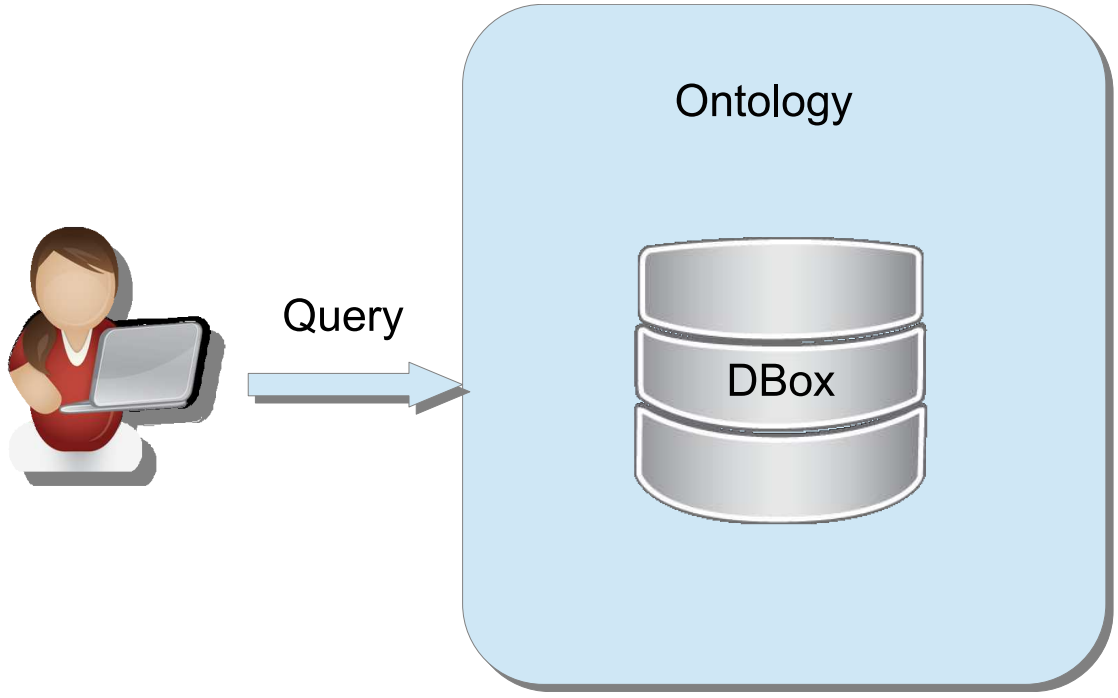
Our Proposal



We propose DBox as a way to model complete data

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Example

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Definition

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- $\{$ `Employee(Jon), Project(Winter)`$\}$ is a model of $\mathcal{D}$
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Certain Answers

Definition

Given:

- an ontology containing: a TBox $\mathcal{T}$, [an ABox $\mathcal{A}$] and
- a DBox $\mathcal{D}$
- a concept query $\mathcal{Q}$ and an individual a

Question: $(\mathcal{T}, [\mathcal{A}], \mathcal{D}) \models \mathcal{Q}(a)$? That is $\forall$ models $\mathcal{I}$ of $(\mathcal{T}, [\mathcal{A}], \mathcal{D})$, is it the case that $\mathcal{I} \models \mathcal{Q}(a)$

Conjunctive Query Answering: DBox vs ABox

- Complexity w.r.t to ABox
  Results by Calvanese, de Giacomo, Lembo, Lenzerini, Rosati, Krisnadhi, Lutz.

Description Logic	KB-type	Combined	Data
$DL-Lite_{[core\|\mathcal{F}]}$	$(\mathcal{T},\mathcal{A})$	NP-complete	in AC^0
$\mathcal{EL}$	$(\mathcal{T},\mathcal{A})$	NP-complete	PTIME-complete

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DBoxes are natural but expensive formalism.

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DBox is expensive

- Do not consider every TBox ⇒ Safe TBox (Lutz & Wolter, 2011)
- Do not consider every query ⇒ Determinacy

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Determinacy

Given a TBox $\mathcal{T}$ and a DBox $\mathcal{D}$, a query $\mathcal{Q}$ is determined by the DBox predicates $\mathbb{P}_{\mathcal{D}}$ if its answer functionally depends only from the extension of the DBox predicates.

Definition (Determinacy)

Let $\mathcal{I}^i_{(\mathcal{D})}$ and $\mathcal{I}^j_{(\mathcal{D})}$ be any two models of a TBox $\mathcal{T}$ embedding a DBox $\mathcal{D}$. A query $\mathcal{Q}$ is *determined* by the DBox predicates $\mathbb{P}_{\mathcal{D}}$ given the TBox $\mathcal{T}$ *if* the answer of $\mathcal{Q}$ over $\mathcal{I}^i_{(\mathcal{D})}$ is the same as the answer of $\mathcal{Q}$ over $\mathcal{I}^j_{(\mathcal{D})}$.

Checking determinacy of a query with a first-order TBox is reducible to entailment.

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Determinacy: Example

Let the DBox predicates be $\mathbb{P}_{\mathcal{D}} = \{\texttt{Mother}, \texttt{Father}\}$ and let $\mathcal{T}$ consists of:

1. $\forall x\ \texttt{Mother(x)} \leftrightarrow \exists y\ \texttt{Woman(x)} \wedge \texttt{hasChild(x,y)}$
2. $\forall x\ \texttt{Father(x)} \leftrightarrow \exists y\ \texttt{Man(x)} \wedge \texttt{hasChild(x,y)}$

and let $\mathcal{Q} = \exists x \exists y ((\texttt{Woman(x)} \vee \texttt{Man(x)}) \wedge \texttt{hasChild(x,y)})$

Fact

$\mathcal{Q}$ is determined by the DBox predicates given the TBox.

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Why Determinacy?

- It captures *exactly* the notion of "non ambiguous" answers.
- Consider the models of a TBox $\mathcal{T}$ with a set of DBox predicates $\mathbb{P}_{\mathcal{D}}$. A query $\mathcal{Q}$ determined by $\mathbb{P}_{\mathcal{D}}$ generates an "extended" DBox augmented with the relation associated to the determined query.
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Theorem (Beth,1953)

Given a TBox $\mathcal{T}$ and a DBox $\mathcal{D}$, a query $\mathcal{Q}$ is determined by the DBox predicates $\mathbb{P}_{\mathcal{D}}$ if and only if there exists an exact reformulation of $\mathcal{Q}$.

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A reformulation $\hat{\mathcal{Q}}$ of a query $\mathcal{Q}$ is exact if $\mathcal{T} \models \forall x.\mathcal{Q}(x) \leftrightarrow \hat{\mathcal{Q}}(x)$.

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Safe-range Queries

- The *safe-range* fragment is a syntactic fragment of FOL
- Intuition: A formula is safe-range if its variables (free and quantified) are bounded by positive predicates or equalities.
  - $\exists x. A(x) \wedge \neg B(x)$ - safe-range
  - $A(x) \vee B(x)$ - safe range
  - $\forall x. C(x)$ - not safe-range
- An open query is *ground safe-range* if its grounding is safe range.
- The safe-range fragment of FOL is equally expressive to the domain independent fragment of FOL and to relational algebra – the core of SQL.

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Query Answering: from entailment to model checking

Theorem

Let $\mathcal{T}$ be an ontology, $\mathcal{Q}$ be a query, $\mathcal{D}$ be a consistent DBox for $\mathcal{T}$, and AD the set of all individuals in $\mathcal{D}$ and $\mathcal{Q}$.
If $\widehat{\mathcal{Q}}$ is

- *an* **exact** *reformulation of $\mathcal{Q}$,*
- **safe-range**,

then

$$Ans(\mathcal{Q}, \mathcal{D}, \mathcal{T}) = Ans(\widehat{\mathcal{Q}}, \mathcal{D}, \{\}) = \{a \mid \langle AD, \mathcal{D}\rangle \models \widehat{\mathcal{Q}}(a)\}$$

The original query answering problem (based on entailment) is reduced to the problem of checking the validity of the reformulation $\widehat{\mathcal{Q}}$ over the *single* interpretation given by the DBox with the active domain (model checking problem), which can be executed by an SQL engine.

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Problem Statment

Given

- an ontology $\mathcal{T}$ in $\mathcal{L}$;
- a DBox $\mathcal{D}$ in $\mathcal{L}$;
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We need to solve the following PROBLEM:

- find a first-order logic **safe-range exact** reformulation of $\mathcal{Q}$ expressed in terms of DBox predicates.

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Semantic Characterisation Theorem

Given a set of database predicates $\mathbb{P}_{\mathcal{DB}}$,
a domain independent ontology $\mathcal{T}$,
and a query $\mathcal{Q}$,

a domain independent exact reformulation $\widehat{\mathcal{Q}}$ of $\mathcal{Q}$ over $\mathbb{P}_{\mathcal{DB}}$ under $\mathcal{T}$ exists

if and only if

$\mathcal{Q}$ is implicitly definable from $\mathbb{P}_{\mathcal{DB}}$ under $\mathcal{T}$
and it is domain independent with respect to $\mathcal{T}$.

Constructive Theorem

If:

1. $\mathcal{T} \cup \widetilde{\mathcal{T}} \models \forall \mathbb{X}.\, \mathcal{Q}_{[\mathbb{X}]} \leftrightarrow \widetilde{\mathcal{Q}}_{[\mathbb{X}]}$
   (that is, $\mathcal{Q}_{[\mathbb{X}]}$ is implicitly definable),
2. $\mathcal{Q}$ is safe-range
   (that is, $\mathcal{Q}$ is domain independent),
3. $\mathcal{T}$ is safe-range
   (that is, $\mathcal{T}$ is domain independent),

then there exists an exact reformulation $\widehat{\mathcal{Q}}$ of $\mathcal{Q}$ as a safe-range query in $\mathcal{FOL}(\mathbb{C}, \mathbb{P})$ over $\mathbb{P}_{\mathcal{DB}}$ under $\mathcal{T}$, that can be obtained constructively.

Oops! But databases are finite

Beth's theorem fails if we consider only models with a finite interpretation of database predicates

Let $\mathbb{P} = \{P, R, A\}$, $\mathbb{P}_{\mathcal{DB}} = \{P, R\}$,
$\mathcal{T}$ consists of:

1. $\forall x, y, z.\, R(x, y) \wedge R(x, z) \rightarrow y = z$,
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3. $(\forall x, y.\, R(x, y) \rightarrow \exists z.\, R(y, z)) \rightarrow (\forall x.\, A(x) \leftrightarrow P(x))$

Fact

- $\forall x, y.\, R(x, y) \rightarrow \exists z.\, R(y, z)$ *is entailed from the first two formulas only over finite interpretations of* R.
- *Query* $\mathcal{Q} = A(x)$ *is finitely determined by DBox predicates, but it is not determined under models with an unrestricted interpretation of* R
- *This knowledge base does not enjoy finitely controllable determinacy.*
- *The characterization theorem fails too.*

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Finitely controllable determinacy

Definition

Given a set of database predicates $\mathbb{P}_{\mathcal{DB}}$, an ontology $\mathcal{T}$, we say that the determinacy of a query $\mathcal{Q}$ is finitely controllable if, $\mathcal{Q}$ is determined iff $\mathcal{Q}$ is determined under the models of $\mathcal{T}$ where the extensions of database predicates are finite.

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Syntax and semantics of $\mathcal{ALCHOIQ}$ concepts and roles

Syntax	Semantics
A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
$\{o\}$	$\{o\}^{I} \subseteq \Delta^{I}$
P	$P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
P^{-}	$\{(y,x) \mid (x,y) \in P^{\mathcal{I}}\}$
$\neg C$	$\Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}$
$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
$\geq nR$	$\{x \mid \#(\{y \mid (x,y) \in R^{\mathcal{I}}\}) \geq n\}$
$\geq nR.C$	$\{x \mid \#(\{y \mid (x,y) \in R^{\mathcal{I}}\} \cap C^{\mathcal{I}}) \geq n\}$

Syntax of $\mathcal{ALCHOIQ}_{GN}$ concepts and roles

$$
\begin{array}{lcl}
R & ::= & P \mid P^- \\
B & ::= & A \mid \{o\} \mid \geq nR \\
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In GN fragments, only guarded negations are allowed.

$\mathcal{ALCHOIQ}_{GN}$

Syntax of $\mathcal{ALCHOIQ}_{GN}$ concepts and roles

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Why does $\mathcal{ALCHOIQ}_{GN}$ make sense ?

- Non-guarded negation should not appear in a cleanly designed ontology, and, if present, it should be fixed.
- The use of absolute negative information:
  "a non-male is a female"
  $\neg$ male $\sqsubseteq$ female;
  is not meaningful in conceptual modelling, since the subsumer includes all sorts of objects in the universe.
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Theorem

The domain independent fragment of $\mathcal{ALCHOIQ}$ and $\mathcal{ALCHOIQ}_{GN}$ are equally expressive.

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A complete procedure

Input: An $\mathcal{ALCHOIQ}_{GN}$ TBox $\mathcal{T}$, a concept query $\mathcal{Q}$ in $\mathcal{ALCHOIQ}_{GN}$, and a database signature (database atomic concepts and roles).

1. Check the implicit definability of the query $\mathcal{Q}$ by testing $\mathcal{T} \cup \widetilde{\mathcal{T}} \models \mathcal{Q} \equiv \widetilde{\mathcal{Q}}$ using a standard OWL2 reasoner ($\mathcal{ALCHOIQ}_{GN}$ is a sub-language of OWL2). If it is the case then continue.
2. Compute the Craig interpolant $\widehat{\mathcal{Q}}(x)$ based on the tableau proof of $((\bigwedge \mathcal{T}) \wedge \mathcal{Q}(x)) \rightarrow ((\bigwedge \widetilde{\mathcal{T}}) \rightarrow \widetilde{\mathcal{Q}}(x))$
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Conclusion and Future Work

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- We introduced a framework to compute an exact reformulation of a concept query under a description logic ontology over some set of concept and role names (DBox predicates).
- We found the conditions which guarantee that a safe-range exact reformulation exists.
- We proved that such safe-range exact reformulation being evaluated as a relational algebra query over the DBox give the same answer as the original query under the ontology.
- An application of the framework to description logics was studied.

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