Revising Horn Theories

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Overview

- Introduction
- (AGM) Belief Revision
- Horn Clause Theories
- Problems with a Naïve Approach to Revision in HC Theories
- Horn Clause Revision
- Conclusions and Future Work

Introduction

The area of *belief change* studies how an agent may change its beliefs in the face of new information.

- Belief change functions include
  - *revision* (where an agent accommodates new information),
  - *contraction* (where an agent's ignorance increases),
  - *merging* (where several agent's knowledge is reconciled),
  - and other operators such as update, forgetting, etc.
- Most work in belief change assumes that the underlying logic subsumes classical PC.
  - More recently there has been work on belief change in weaker systems
  - E.g. belief change in DLs, contraction in Horn theories.

Horn Theory Revision

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Why?

- Agents will change their beliefs.
  - It is crucial to have a comprehensive theory of belief change.
  - Work on inferentially weak approaches sheds light on the foundations of belief change.
- Horn clauses are employed in areas such as AI, DB, and LP.
- While Horn contraction has been studied, Horn contraction doesn't seem to help wrt defining revision.

Introduction: Belief Revision

Example

Informally, we have an agent, and some new piece of information that is to be incorporated into the agent's set of beliefs.

Beliefs: The person with the coffee mug is a teaching assistant.
The person with the coffee mug is a Ph.D. student.
Ph.D. students are graduate students.
Graduate students who are teaching assistants can't hold university fellowships.

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New Information: The person with the coffee mug has a fellowship.

☞ In this case, the new information conflicts with the agent's beliefs.

Belief Revision

In belief revision, an agent

- incorporates a new belief ϕ, while
- maintaining consistency (unless $\vdash \neg\phi$).

Thus an agent may have to remove beliefs to remain consistent.

Problem: Logical considerations alone are not sufficient to determine a revision function.

- But there are general principles that should be shared by all revision functions. (E.g. $\phi \in K * \phi$.)

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Belief Sets: Describe belief change at the knowledge level, on an abstract level, independent of how beliefs are represented.

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Belief Bases: A knowledge base is an arbitrary set of formulas

Example

$$K_1 = \{p, q\} \qquad K_2 = \{p, p \supset q\}$$

A belief base approach would distinguish these KBs.
A belief set approach does not.

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Postulates: Criteria that should bound any "rational" function.

- E.g. If $\not\vdash \phi$ then $\phi \notin K - \phi$.

Ideally: Show that a construction $\approx$ a postulate set.

- E.g. the AGM contraction postulates exactly capture remainder-set contraciton.

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- The preorder gives the plausibility of a interpretation wrt K, and can be taken as specifying an agent's epistemic state.
- Define: $Mod(K * \phi) \; = \; \min(Mod(\phi), \preceq_K)$.
- I.e. the revision of K by ϕ is characterized by the most plausible ϕ worlds according to the agent.

AGM Revision Postulates

The AGM Postulates are the best-known set for revision.

- (K*1) $K * \phi = \mathcal{C}n(K * \phi)$
- (K*2) $\phi \in K * \phi$
- (K*3) $K * \phi \subseteq K + \phi$
- (K*4) If $\neg\phi \notin K$ then $K + \phi \subseteq K * \phi$
- (K*5) $K * \phi$ is inconsistent only if ϕ is inconsistent
- (K*6) If $\phi \equiv \psi$ then $K * \phi = K * \psi$
- (K*7) $K * (\phi \wedge \psi) \subseteq K * \phi + \psi$
- (K*8) If $\neg\psi \notin K * \phi$ then $K * \phi + \psi \subseteq K * (\phi \wedge \psi)$

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☞ These postulates exactly capture revision defined in terms of faithful assignments.

Horn Clauses

Preliminaries:

- P is a finite set of propositional variables.
- $a_1 \wedge a_2 \wedge \cdots \wedge a_n \to a$ is a *Horn clause*, where $n \geq 0$ and a, $a_i \in P \cup \{\bot\}$ for $1 \leq i \leq n$.
  - If $n = 0$ then $\to a$ is also written a, and is a *fact*.
- A *Horn formula* is a conjunction of Horn clauses.
- $\mathcal{L}_H$ is the language of Horn formulas.

☞ Henceforth we'll deal exclusively with Horn formulas.

Horn Clauses (cont'd)

- An *interpretation* m is identified with a subset of P.
  - On occasion we will list negated atoms or use juxtaposition.
  - E.g. for $P = \{p, q\}$, interpretation $\{p\}$ may be written $\{p, \neg q\}$ or $p\overline{q}$.
- Notions of truth, entailment, etc. carry over from classical logic.
- $\vdash$ can be defined strictly in terms of Horn formulas.

Horn Clauses (cont'd)

Key Fact:

> *Models of Horn formulas are closed under intersection of positive atoms.*

That is:

> *If m_1, $m_2 \in Mod(\phi)$ then $m_1 \cap m_2 \in Mod(\phi)$.*

E.g. For $P = \{p, q, r\}$,
$Mod(\neg p \vee \neg q) = \{pr, qr, r, p, q, \emptyset\}$.

Horn Theory Revision

Goal: Characterize $H' = H * \phi$ where H, H' are HC belief sets and ϕ is a conjunction of Horn clauses (= *Horn formula*).

- This will be done within the framework of Horn logic.
- So the formal development makes no reference to classical PC.

Aside: Horn Theory Contraction

- The case of Horn contraction was worked out in [D, KR08], [D&W, KR10], [Z&P, IJCAI11].
- Key Problem: Horn remainder sets aren't adequate for capturing contraction.
- I.e. a contraction $H - \phi$ can't be fully specified in terms of maximal (Horn) subsets of H that fail to imply ϕ.

☞ This indicates that there may similarly be problems for Horn revision.

Applying AGM to Horn

Let $(H * 1)$–$(H * 8)$ stand for the AGM postulates expressed in terms of Horn theories and Horn formulas.

☞ If we try to define Horn revision in terms of faithful rankings and $(H * 1)$–$(H * 8)$, we run into problems.

Horn Theory Revision: Problems

Interdefinability results between revision and contraction don't hold.

- In AGM approach, can define revision in terms of contraction by:

$$K * \phi \ = \ (K - \neg\phi) + \phi$$

- Problem: $\neg\phi$ may not be Horn.
- As well, there are other problems [D, KR08].

HC Revision: Problem 2

Distinct rankings may yield the same revision function.

- Let $P = \{p, q\}$; consider the three total preorders:

$$pq \prec \overline{pq} \prec p\overline{q} \prec \overline{p}q$$
$$pq \prec \overline{pq} \prec \overline{p}q \prec p\overline{q}$$
$$pq \prec \overline{pq} \prec \overline{p}q \approx p\overline{q}$$

- These rankings yield the same revision function.
- Informally, can't distinguish $\overline{p}q$ and $p\overline{q}$ via Horn clauses.
- Problem: Rankings may be *underconstrained* by Horn AGM postulates.

HC Revision: Problem 3

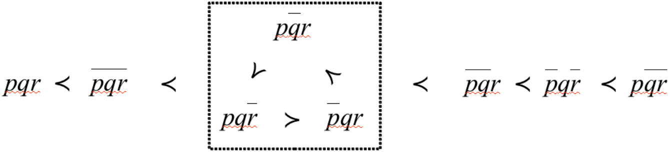
Some postulates may not be satisfied in a faithful ranking.

- See [D&P, IJCAI11] for an example that violates (H*7) and (H*8).
- Problem: There are sets of interpretations for which there is no corresponding Horn theory.

HC Revision: Problem 4

There are Horn AGM revision functions that cannot be modelled by preorders on interpretations.

- A revision function defined in terms of the following pseudo-preorder satisfies the Horn revision postulates.



- Problem: The postulates are too weak to rule out some undesirable non-preorders.

Horn Theory Revision: Solution

To address these problems we

- add a condition to restrict faithful rankings; and
- add a postulate to the set of Horn AGM postulates.

Note:

- In propositional logic, these additions are redundant.
- Hence, our solution is a *generalization* of AGM revision.

Horn Theory Revision: Ranking Functions

We restrict faithful rankings to *Horn compliant* rankings.

Definition

- A set of W interpretations is *Horn elementary* iff there is a Horn formula ϕ such that $W = Mod(\phi)$.
- A preorder $\preceq_H$ is *Horn compliant* iff for every formula $\phi \in \mathcal{L}_H$, $\min(Mod(\phi), \preceq_H)$ is Horn elementary.

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Subtlety:

- Equivalently-ranked interpretations in a Horn compliant ranking may not be Horn elementary.

Horn Theory Revision: New Postulate

We add the schema:

(Acyc) If for $0 \leq i < n$ we have $(H * \mu_{i+1}) + \mu_i \not\vdash \bot$, and $(H * \mu_0) + \mu_n \not\vdash \bot$, then $(H * \mu_n) + \mu_0 \not\vdash \bot$.

Intuition: $(H * \mu_{i+1}) + \mu_i \not\vdash \bot$ holds if the least μ_i interpretations in a ranking are not greater than the least μ_{i+1} interpretations.

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We have:

- Acyc is a logical consequence of the AGM postulates in PC.
- Acyc is independent of the Horn AGM postulates.

Representation Result

These changes prove sufficient for capturing Horn revision:

Theorem:
A revision operator $*$ satisfies (H*1) – (H*8) and (Acyc)

iff

there is a faithful assignment that maps each Horn belief set H to a total preorder $\preceq_H$ such that $\preceq_H$ is Horn compliant and

$$Mod(H * \phi) = \min(Mod(\phi), \preceq_H)$$

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  - As noted, Horn contraction has problems analogous to Horn revision wrt characterizations.

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- It isn't clear how to link Horn revision and contraction.
  - Horn revision and contraction seem to be quite distinct operations.
  - As noted, Horn contraction has problems analogous to Horn revision wrt characterizations.
- Moreover answers to these questions are important for principled approaches to change in (inferentially-weak) systems.

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- ... while being applicable in areas like AI, DB, and LP.