The Suppression Task

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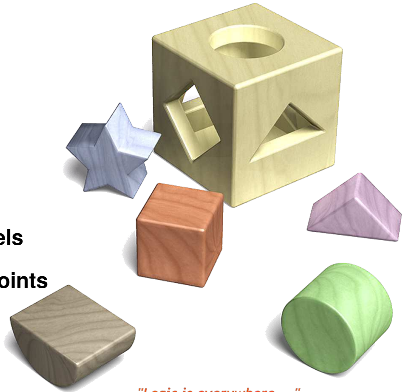
- The Suppression Task – Part I
- Three-Valued Łukasiewicz Logic
- Logic Programs
- Completion Semantics and Least Models
- Semantic Operators and Least Fixed Points
- Contractions
- The Suppression Task – Part II
- Abduction



"Logic is everywhere ..."

The Suppression Task – Forward Reasoning

- Byrne: Suppressing Valid Inferences with Conditionals.
  Cognition 31, 61-83: 1989
- If she has an essay to write then she will study late in the library.
  She has an essay to write.
  - Modus Ponens (MP) in classical logic
  - 96% of subjects conlude that she will study late in the library.
- If she has an essay to write then she will study late in the library.
  She has an essay to write.
  If she has a textbook to read she will study late in the library.
  - Alternative Arguments
  - 96% of subjects conlude that she will study late in the library.
- If she has an essay to write then she will study late in the library.
  She has an essay to write.
  If the library stays open she will study late in the library
  - 38% of subjects conlude that she will study late in the library.
  - Additional arguments lead to suppression of earlier conclusions.

Reasoning Towards an Appropriate Logical Form

- **Context independent rules**
  - **If she has an essay to write and the library is open then she will study late in the library.
    If the library is open and she has a reason for studying in the library then she will study late in the library.**
- **Context dependent rule plus exception**
  - **If she has an essay to write then she will study late in the library.
    However, if the library is not open, then she will not study late in the library.**
  - **The last sentence is the contrapositive of the converse of the original sentence!**

The Suppression Task – Denial of Antecedent (DA)

- Byrne: Suppressing Valid Inferences with Conditionals.
  Cognition 31, 61-83: 1989
- **If she has an essay to write then she will study late in the library.**
  She does not have an essay to write.
  - **46% of subjects conlude that she will not study late in the library.**
- **If she has an essay to write then she will study late in the library.**
  She does not have an essay to write.
  If she has a textbook to read she will study late in the library.
  - **4% of subjects conlude that she will not study late in the library.**
- **If she has an essay to write then she will study late in the library.**
  She does not have an essay to write.
  If the library stays open she will study late in the library.
  - **63% of subjects conlude that she will not study late in the library.**

Human Reasoning – The Search for Models

- **Goal** **Find a logic which adequately models human reasoning.**
- **How about classical two-valued propositional logic?**
- **Let's consider a direct encoding:**

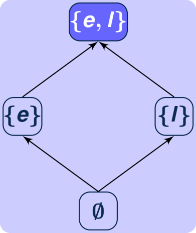
$$\{l \leftarrow e,\ e\}$$
$$\{l \leftarrow e,\ e,\ l \leftarrow t\}$$
$$\{l \leftarrow e,\ e,\ l \leftarrow o\}$$

$$\{l \leftarrow e,\ \neg e\}$$
$$\{l \leftarrow e,\ \neg e,\ l \leftarrow t\}$$
$$\{l \leftarrow e,\ \neg e,\ l \leftarrow o\}$$

Two-Valued Interpretations

- Let $\mathcal{L}$ be a language of propositional logic.
- A (two-valued) interpretation is a mapping $\mathcal{L} \mapsto \{\top, \bot\}$ represented by I, where I is a set containing all atoms which are mapped to $\top$.
  - All atoms which do not occur in I are mapped to $\bot$.
- Let $\mathcal{I}$ denote the set of all interpretations.
  - $(\mathcal{I}, \subseteq)$ is a lattice.
- An interpretation I is a model for a program $\mathcal{P}$, in symbols $I \models \mathcal{P}$, iff $I(\mathcal{P}) = \top$.

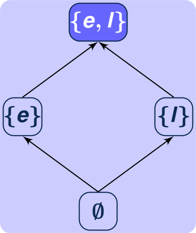
$$
\begin{array}{rcl}
\emptyset & \not\models & \{l \leftarrow e,\ e\} \\
\{e\} & \not\models & \{l \leftarrow e,\ e\} \\
\{l\} & \not\models & \{l \leftarrow e,\ e\} \\
\{e, l\} & \models & \{l \leftarrow e,\ e\}
\end{array}
$$



Logical Consequence (1)

- A formula G is a logical consequence of a set of formulas $\mathcal{F}$, in symbols $\mathcal{F} \models G$, iff all models for $\mathcal{F}$ are also models for G.

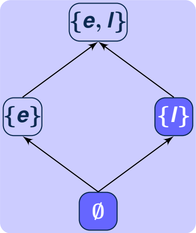
$$\{l \leftarrow e,\ e\} \models l$$
$$\{l \leftarrow e,\ e\} \models e$$



Logical Consequence (2)

- A formula G is a logical consequence of a set of formulas $\mathcal{F}$, in symbols $\mathcal{F} \models G$, iff all models for $\mathcal{F}$ are also models for G.

$$\{l \leftarrow e,\ \neg e\} \models \neg e$$
$$\{l \leftarrow e,\ \neg e\} \not\models l$$
$$\{l \leftarrow e,\ \neg e\} \not\models \neg l$$



The Suppression Task – A Classical Logic Approach

- **Recall the examples:**

$\{l \leftarrow e,\ e\}$	$\models$	l	**modus ponens**
$\{l \leftarrow e,\ e,\ l \leftarrow t\}$	$\models$	l	**classical logic is monoton**
$\{l \leftarrow e,\ e,\ l \leftarrow o\}$	$\models$	l	**upps, humans don't do this**
$\{l \leftarrow e,\ \neg e\}$	$\not\models$	$\neg l$	**denial of antecendent**
$\{l \leftarrow e,\ \neg e,\ l \leftarrow t\}$	$\not\models$	$\neg l$	
$\{l \leftarrow e,\ \neg e,\ l \leftarrow o\}$	$\not\models$	$\neg l$	

- **Conclusion classical logic is inadequate.**
  - **Often mistakenly generalized to "logic is inadequate".**

The Suppression Task – A Computational Logic Approach

- **Goal** Find a logic which adequately models human reasoning.
- **Solution** I propose the following:
  - Logic programs under (weak) completion semantics
    - Non-monotonicity
  - Reasoning towards an appropriate logical form
    - Logic programs
  - Three-valued Łukasiewicz logic
    - Least models
  - An appropriate semantic operator
    - Least fixed points are least models
    - Least fixed points can be computed by iterating the operator
  - Reasoning with respect to the least models
  - A connectionist realization

Adequateness

- **When is a logic adequate?**
- **In this talk** **If it qualitatively gives the same answers as subjects in the corresponding experiments.**

Logic Programs

- A **(logic) program** is a finite set of clauses.
  - A **(program) clause** is an expression of the form $A \leftarrow B_1 \wedge \cdots \wedge B_n$, where $n \geq 1$, A is an atom, and each B_i, $1 \leq i \leq n$, is either a literal, $\top$ or $\bot$.
  - A is called **head** and $B_1 \wedge \cdots \wedge B_n$ **body** of the clause.
  - A clause of the form $A \leftarrow \top$ is called a **positive fact**.
  - A clause of the form $A \leftarrow \bot$ is called a **negative fact**.

$$\{l \leftarrow e,\ e \leftarrow \top\}$$
$$\{l \leftarrow e,\ e \leftarrow \top,\ l \leftarrow t\}$$
$$\{l \leftarrow e,\ e \leftarrow \bot\}$$

- $\mathcal{P}$ is **definite** if the bodies of all clauses of $\mathcal{P}$ consist only of atoms and $\top$.
- Here I consider only propositional programs,
  but the approach extends to first-order programs.
- The language $\mathcal{L}$ underlying a program $\mathcal{P}$ shall contain precisely
  the relation symbols occurring in $\mathcal{P}$, and no others.

Program Completion

- Let $\mathcal{P}$ be a program. Consider the following transformation:

  1. All clauses with the same head $A \leftarrow Body_1,\ A \leftarrow Body_2, \ldots$ are replaced by $A \leftarrow Body_1 \vee Body_2 \vee \ldots$.
  2. If an atom A is not the head of any clause in $\mathcal{P}$ then add $A \leftarrow \bot$.
  3. All occurrences of $\leftarrow$ are replaced by $\leftrightarrow$.

  The resulting set is called completion of $\mathcal{P}$ or c $\mathcal{P}$.

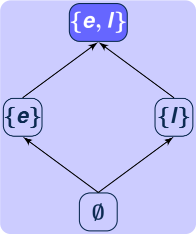
  If 2 is omitted then the resulting set is called weak completion of $\mathcal{P}$ or wc $\mathcal{P}$.

Program Completion – Example 1

- **Consider**

$$\begin{array}{lcl} \mathcal{P}_1 & = & \{l \leftarrow e,\ e \leftarrow \top\} \\ \mathrm{c}\,\mathcal{P}_1 & = & \{l \leftrightarrow e,\ e \leftrightarrow \top\} \\ \mathrm{wc}\,\mathcal{P}_1 & = & \{l \leftrightarrow e,\ e \leftrightarrow \top\} \end{array}$$

- **The only model of $\mathrm{c}\,\mathcal{P}_1$ and $\mathrm{wc}\,\mathcal{P}_1$ is:**



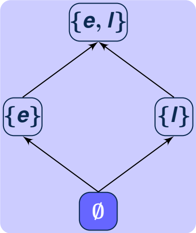
- **Hence, $\mathrm{c}\,\mathcal{P}_1 \models l$ and $\mathrm{wc}\,\mathcal{P}_1 \models l$.**

Program Completion – Example 2

- Consider

$$
\begin{array}{lcl}
\mathcal{P}_2 & = & \{l \leftarrow e,\ e \leftarrow \bot\} \\
c\,(\mathcal{P}_2) & = & \{l \leftrightarrow e,\ e \leftrightarrow \bot\} \\
wc\,(\mathcal{P}_2) & = & \{l \leftrightarrow e,\ e \leftrightarrow \bot\}
\end{array}
$$

- The only model of c $\mathcal{P}_2$ and wc $\mathcal{P}_2$ is:



- Hence, c $\mathcal{P}_2 \models \neg l$ and wc $\mathcal{P}_2 \models \neg l$.
- Remember, $\mathcal{P}_2 \not\models \neg l$.

Program Completion – Example 3

- Consider

$$\begin{array}{lcl} \mathcal{P}_3 & = & \{l \leftarrow e,\ e \leftarrow \top,\ l \leftarrow t\} \\ \mathrm{c}\,\mathcal{P}_3 & = & \{l \leftrightarrow e \vee t,\ e \leftrightarrow \top,\ t \leftrightarrow \bot\} \\ \mathrm{wc}\,\mathcal{P}_3 & = & \{l \leftrightarrow e \vee t,\ e \leftrightarrow \top\} \end{array}$$

- The only model of $\mathrm{c}\,\mathcal{P}_3$ is:

$$\{e, l\}$$

- The models of $\mathrm{wc}\,\mathcal{P}_3$ are:

$$\{e, l\}$$
$$\{e, l, t\}$$

- Hence, $\mathrm{c}\,\mathcal{P}_3 \models \neg t$ whereas $\mathrm{wc}\,\mathcal{P}_3 \not\models \neg t$ and $\mathrm{wc}\,\mathcal{P}_3 \not\models t$.

Monotonicity

- Let $\mathcal{F}$ and $\mathcal{F}'$ be sets of formulas and G a formula.
  A logic is monotonic if the following holds:
  If $\mathcal{F} \models G$ then $\mathcal{F} \cup \mathcal{F}' \models G$.
- Classical logic is monotonic.
- A logic based on completion semantics is non-monotonic.
  - Consider
$$\begin{array}{lcl} \mathcal{P}_3 & = & \{l \leftarrow e,\ e \leftarrow \top,\ l \leftarrow t\} \\ \mathcal{P}_3' & = & \mathcal{P} \cup \{t \leftarrow \top\} \end{array}$$
  - Then
$$\begin{array}{lcl} \mathrm{c}\,\mathcal{P}_3 & \models & \neg t \\ \mathrm{c}\,\mathcal{P}_3' & \not\models & \neg t \end{array}$$

Reasoning Towards an Appropriate Logical Form (1)

- Stenning, van Lambalgen: Human Reasoning and Cognititve Science. MIT Press: 2008
- **Represent conditionals as licences for conditionals.**
  - If she has an essay to write then she will study late in the library.
    She has an essay to write.
    - $\mathcal{P}_4 = \{l \leftarrow e \wedge \neg ab,\ ab \leftarrow \bot,\ e \leftarrow \top\}$
  - If she has an essay to write then she will study late in the library.
    She has an essay to write.
    If she has a textbook to read she will study late in the library.
    - $\mathcal{P}_5 = \{l \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ l \leftarrow t \wedge \neg ab_2,\ ab_2 \leftarrow \bot,\ e \leftarrow \top\}$
- **Reason about additional premises.**
  - If she has an essay to write then she will study late in the library.
    She has an essay to write.
    If the library stays open she will study late in the library.
    - $\mathcal{P}_6 = \{l \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \neg o,\ l \leftarrow o \wedge \neg ab_2,\ ab_2 \leftarrow \neg e,\ e \leftarrow \top\}$

Reasoning Towards an Appropriate Logical Form (2)

- **Denial of Antecedent (DA)**
  - If she has an essay to write then she will study late in the library.
    She does not have an essay to write.
    - $\mathcal{P}_7 = \{l \leftarrow e \wedge \neg ab,\ ab \leftarrow \bot,\ e \leftarrow \bot\}$
  - If she has an essay to write then she will study late in the library.
    She does not have an essay to write.
    If she has a textbook to read she will study late in the library.
    - $\mathcal{P}_8 = \{l \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ l \leftarrow t \wedge \neg ab_2,\ ab_2 \leftarrow \bot,\ e \leftarrow \bot\}$
  - If she has an essay to write then she will study late in the library.
    She does not have an essay to write.
    If the library stays open she will study late in the library.
    - $\mathcal{P}_9 = \{l \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \neg o,\ l \leftarrow o \wedge \neg ab_2,\ ab_2 \leftarrow \neg e,\ e \leftarrow \bot\}$

Three-Valued Logics

- Consider the following truth table

		$\neg$	$\wedge$	$\vee$	$\leftarrow_Ł$	$\leftrightarrow_Ł$	$\leftarrow_K$	$\leftrightarrow_K$	$\leftrightarrow_C$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$	$\bot$
$\top$	U	$\bot$	U	$\top$	$\top$	U	$\top$	U	$\bot$
$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$
$\bot$	U	$\top$	$\bot$	U	U	U	U	U	$\bot$
U	$\top$	U	U	$\top$	U	U	U	U	$\bot$
U	$\bot$	U	$\bot$	U	$\top$	U	$\top$	U	$\bot$
U	U	U	U	U	$\top$	$\top$	U	U	$\top$

- Different three-valued logics

Łukasiewicz (Ł) semantics 1920	$\neg$	$\wedge$	$\vee$	$\leftarrow_Ł$	$\leftrightarrow_Ł$
Kleene (K) semantics 1952	$\neg$	$\wedge$	$\vee$	$\leftarrow_K$	$\leftrightarrow_K$
Fitting (F) semantics 1985	$\neg$	$\wedge$	$\vee$	$\leftarrow_K$	$\leftrightarrow_C$

$$(F \leftrightarrow G) \left\{ \begin{array}{c} \equiv_{3Ł} \\ \not\equiv_{3F} \end{array} \right\} (F \leftarrow G) \wedge (G \leftarrow F)$$

Principles of Łukasiewicz

- **Łukasiewicz used 1, .5, and 0 instead of $\top$, U, and $\bot$, respectively.**
- **Principles of identity and non-identity**
  - $(\bot \leftrightarrow \bot) \equiv (\top \leftrightarrow \top) \equiv (U \leftrightarrow U) \equiv \top$.
  - $(\top \leftrightarrow \bot) \equiv (\bot \leftarrow \top) \equiv \bot$.
  - $(\bot \leftrightarrow U) \equiv (U \leftrightarrow \bot) \equiv (\top \leftrightarrow U) \equiv (U \leftrightarrow \top) \equiv U$.
- **Principles of implication**
  - $(\bot \leftarrow \bot) \equiv (\top \leftarrow \top) \equiv (U \leftarrow U) \equiv \top$.
    $(\top \leftarrow \bot) \equiv (\top \leftarrow U) \equiv (U \leftarrow \bot) \equiv \top$.
  - $(\bot \leftarrow \top) \equiv \bot$.
  - $(\bot \leftarrow U) \equiv (U \leftarrow \top) \equiv U$.
- **Definitions of negation, disjunction, and conjunction**
  - $\neg F \equiv (\bot \leftarrow F)$
  - $(F \vee G) \equiv (G \leftarrow (G \leftarrow F))$.
  - $(F \wedge B) \equiv \neg(\neg F \vee \neg G)$.

Some Common Laws and Literature

- **Laws**

		Ł	K	F
Equivalence	$F \leftrightarrow G \equiv (F \leftarrow G) \wedge (G \leftarrow F)$	yes	yes	no
Implication	$F \rightarrow G \equiv \neg F \vee G$	no	yes	yes
Syllogism	$(F \rightarrow G) \wedge (G \rightarrow H) \equiv F \rightarrow H$	no	yes	yes
Excluded Middle	$F \vee \neg F \equiv \top$	no	no	no
Contradiction	$F \wedge \neg F \equiv \bot$	no	no	no

- **Literature**

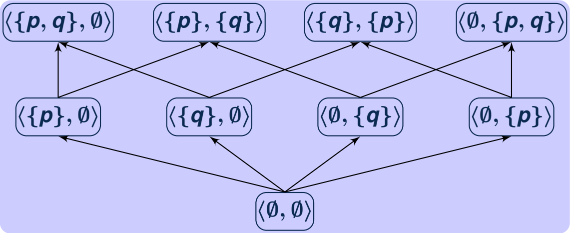
  - Fitting: A Kripke-Kleene Semantics for Logic Programs. Journal of Logic Programming 2, 295-312: 1985.
  - Kleene: Introduction to Metamathematics. North-Holland: 1952.
  - Łukasiewicz: O logice trójwartościowej. Ruch Filozoficzny 5, 169-171: 1920. English translation: On Three-Valued Logic. In: *Jan Łukasiewicz Selected Works*. (L. Borkowski, ed.), North Holland, 87-88, 1990.

Three-Valued Interpretations

- A (three-valued) interpretation is a mapping $\mathcal{L} \mapsto \{\top, \bot, \mathsf{U}\}$ represented by $\langle I^\top, I^\bot \rangle$, where
  - $I^\top$ contains all atoms which are mapped to $\top$,
  - $I^\bot$ contains all atoms which are mapped to $\bot$,
  - $I^\top \cap I^\bot = \emptyset$.
  - All atoms which occur neither in $I^\top$ nor $I^\bot$ are mapped to U.

Three-Valued Interpretations and Models

- Let $\mathcal{I}$ denote the set of all interpretations.
  - Fitting 1985 $(\mathcal{I}, \subseteq)$ is a complete semi-lattice.



- An interpretation I is a model for a program $\mathcal{P}$, in symbols $I \models_3 \mathcal{P}$, iff $I(\mathcal{P}) = \top$.

$$\langle\emptyset, \emptyset\rangle \left\{ \begin{array}{c} \models_{3Ł} \\ \not\models_{3F} \end{array} \right\} \{p \leftarrow q\}$$

Logic Programs under Three-Valued Ł-Semantics

- We consider Łukasiewicz semantics.
- Let $\mathcal{P}$ be a logic program and $I = \langle I^\top, I^\bot \rangle$ be an interpretation.
- **Proposition 1** If $I = \langle I^\top, I^\bot \rangle \models_{3Ł} \mathcal{P}$ then $I' = \langle I^\top, \emptyset \rangle \models_{3Ł} \mathcal{P}$.
- **Proof** Suppose $I = \langle I^\top, I^\bot \rangle \models_{3Ł} \mathcal{P}$
  - Let $A \leftarrow Body \in \mathcal{P}$.
  - **To show** $I' \models_{3Ł} A \leftarrow Body$.
  - We distinguish three cases
    1. $A \in I^\top$ In this case, $I' \models_{3Ł} A \leftarrow Body$.
    2. $A \in I^\bot$
    3. $A \notin I^\top \cup I^\bot$

Proof of Proposition 1 Case 2

- $A \in I^{\perp}$ In this case $I(A) = \bot$ and $I'(A) = \text{U}$.
  - Because $I \models_{3Ł} A \leftarrow Body$ we conclude $I(Body) = \bot$.
  - Hence, we find $L \in Body$ such that $I(L) = \bot$.
    - $L = B$ In this case $I(B) = \bot$ and, hence, $I'(B) = I'(L) = \text{U}$.
    - $L = \neg B$ In this case $I(B) = \top$ and, hence, $I'(B) = \top$ and $I'(L) = \bot$.
  - Consequently, $I'(Body) \in \{\text{U}, \bot\}$.
  - Because $I'(A) = \text{U}$ we conclude $I \models_{3Ł} A \leftarrow Body$.

Proof of Proposition 1 Case 3

- $A \not\in I^\top \cup I^\bot$ In this case $I(A) = I'(A) = \mathsf{U}$.
  - $I(Body) = \bot$ As in the previous case we find $I'(Body) \in \{\bot, \mathsf{U}\}$
    - Consequently, $I' \models_{3Ł} A \leftarrow Body$.
  - $I(Body) = \mathsf{U}$ In this case we find $L \in Body$ with $I(L) = \mathsf{U}$.
    - Then, $I'(L) = \mathsf{U}$.
    - Consequently, $I'(Body) = \mathsf{U}$.
    - Hence, $I' \models_{3Ł} A \leftarrow Body$. qed
- Example Consider $\mathcal{P} = \{p \leftarrow q \wedge \neg r\}$.
  - $\langle\{p, q\}, \{r\}\rangle$ is a model for $\mathcal{P}$, and so is $\langle\{p, q\}, \emptyset\rangle$.
  - $\langle\{p, r\}, \{q\}\rangle$ is a model for $\mathcal{P}$, and so is $\langle\{p, r\}, \emptyset\rangle$.
  - $\langle\{r\}, \{q\}\rangle$ is a model for $\mathcal{P}$, and so is $\langle\{r\}, \emptyset\rangle$.

Intersection of Two Models

- We consider Łukasiewicz semantics; let $\mathcal{P}$ be a logic program.
- **Proposition 2** Let $I_1 = \langle I_1^\top, \emptyset\rangle$ and $I_2 = \langle I_2^\top, \emptyset\rangle$ be two models of $\mathcal{P}$. Then, $I_3 = \langle I_1^\top \cap I_2^\top, \emptyset\rangle$ is also a model for $\mathcal{P}$.
- **Proof** Suppose $I_3 \not\models_{3Ł} \mathcal{P}$.
  - Then, we find $A \leftarrow Body \in \mathcal{P}$ such that $I_3(A \leftarrow Body) \neq \top$.
  - We distinguish the following cases:
    1. $I_3(A) = \bot$ and $I_3(Body) = \top$ Impossible, because $I_3^\bot = \emptyset$.
    2. $I_3(A) = \bot$ and $I_3(Body) = U$ Impossible, because $I_3^\bot = \emptyset$.
    3. $I_3(A) = U$ and $I_3(Body) = \top$

       We find $j \in \{1, 2\}$ with $I_j(A) = U$.

       Because $I_j \models_{3Ł} A \leftarrow Body$ we find $I_j(Body) \in \{U, \bot\}$. (∗)

       Because $I_3(Body) = \top$ and $I_3^\bot = \emptyset$ we find
       for all $L \in Body$ that L is an atom and $L \in I_3^\top$.

       Hence, for all $L \in Body$ we find $L \in I_j^\top$, $j \in \{1, 2\}$.

       Consequently, $I_j(Body) = \top$, $j \in \{1, 2\}$ contradicting (∗) qed

Intersection of Two Models – Example

- Proposition 2 does not hold for arbitrary models.
  - Consider $\mathcal{P} = \{p \leftarrow q_1 \wedge r_1,\ p \leftarrow q_2 \wedge r_2\}$.
  - Let $I_1 = \langle \emptyset, \{p, q_1, r_2\} \rangle$ and $I_2 = \langle \emptyset, \{p, q_2, r_1\} \rangle$.
  - Both, I_1 and I_2, are models for $\mathcal{P}$.
  - However, $I_1 \cap I_2 = \langle \emptyset, \{p\} \rangle$ is not a model for $\mathcal{P}$.

Model Intersection

- We consider Łukasiewicz semantics; let $\mathcal{P}$ be a logic program.
- **Theorem 3** The model intersection property holds for $\mathcal{P}$,
  i.e., $\bigcap\{I \mid I \models_{3Ł} \mathcal{P}\} \models_{3Ł} \mathcal{P}$.
- **Proof** Follows immediately from Propositions 1 and 2 qed
- Let $lm_{3Ł}\,\mathcal{P}$ denote the least model of $\mathcal{P}$ under Łukasiewicz semantics.

$$lm_{3Ł}\,\{p \leftarrow q\} = \langle\emptyset, \emptyset\rangle$$

- **Observation** Theorem 3 does not hold under Fitting semantics:

$$\begin{array}{lll} \langle\{p, q\}, \emptyset\rangle & \models_{3F} & \{p \leftarrow q\} \\ \langle\emptyset, \{p, q\}\rangle & \models_{3F} & \{p \leftarrow q\} \end{array} \quad \text{but} \quad \langle\emptyset, \emptyset\rangle \not\models_{3F} \{p \leftarrow q\}$$

Weakly Completed Logic Programs under Ł-Semantics

- **We consider Łukasiewicz semantics.**
- **Let $\mathcal{P}$ be a logic program.**
- **Theorem 4** **The model intersection property holds for wc $\mathcal{P}$ as well.**
- **Theorem 5** **If $I \models_{3Ł}$ wc $\mathcal{P}$ then $I \models_{3Ł} \mathcal{P}$.**
- **Observation** **Theorem 5 does not hold under F-semantics.**

$$\langle \emptyset, \emptyset \rangle \models_{3F} \text{wc}\, \{p \leftarrow q\} = \{p \leftrightarrow q\}, \text{ but } \langle \emptyset, \emptyset \rangle \not\models_{3F} \{p \leftarrow q\}$$

Reasoning with Respect to the Least Model of wc $\mathcal{P}$

- **Recall our examples**

$$
\begin{array}{lcl}
\text{wc } \mathcal{P}_4 & = & \{l \leftrightarrow e \wedge \neg ab,\ ab \leftrightarrow \bot,\ e \leftrightarrow \top\} \\
\text{Im}_{3Ł} \text{ wc } \mathcal{P}_4 & = & \langle \{e, l\}, \{ab\} \rangle \ \rightsquigarrow\ \text{Im}_{3Ł} \text{ wc } \mathcal{P}_4(l) = \top \\
\text{wc } \mathcal{P}_5 & = & \{l \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2),\ ab_1 \leftrightarrow \bot,\ ab_2 \leftrightarrow \bot,\ e \leftrightarrow \top\} \\
\text{Im}_{3Ł} \text{ wc } \mathcal{P}_5 & = & \langle \{e, l\}, \{ab_1, ab_2\} \rangle \ \rightsquigarrow\ \text{Im}_{3Ł} \text{ wc } \mathcal{P}_5(l) = \top \\
\text{wc } \mathcal{P}_6 & = & \{l \leftrightarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_2),\ ab_1 \leftrightarrow \neg o,\ ab_2 \leftrightarrow \neg e,\ e \leftrightarrow \top\} \\
\text{Im}_{3Ł} \text{ wc } \mathcal{P}_6 & = & \langle \{e\}, \{ab_2\} \rangle \ \rightsquigarrow\ \text{Im}_{3Ł} \text{ wc } \mathcal{P}_6(l) = \mathsf{U} \\
\text{wc } \mathcal{P}_7 & = & \{l \leftrightarrow e \wedge \neg ab,\ ab \leftrightarrow \bot,\ e \leftrightarrow \bot\} \\
\text{Im}_{3Ł} \text{ wc } \mathcal{P}_7 & = & \langle \emptyset, \{e, l, ab\} \rangle \ \rightsquigarrow\ \text{Im}_{3Ł} \text{ wc } \mathcal{P}_7(l) = \bot \\
\text{wc } \mathcal{P}_8 & = & \{l \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2),\ ab_1 \leftrightarrow \bot,\ ab_2 \leftrightarrow \bot,\ e \leftrightarrow \bot\} \\
\text{Im}_{3Ł} \text{ wc } \mathcal{P}_8 & = & \langle \emptyset, \{e, ab_1, ab_2\} \rangle \ \rightsquigarrow\ \text{Im}_{3Ł} \text{ wc } \mathcal{P}_8(l) = \mathsf{U} \\
\text{wc } \mathcal{P}_9 & = & \{l \leftarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_2),\ ab_1 \leftrightarrow \neg o,\ ab_2 \leftrightarrow \neg e,\ e \leftrightarrow \bot\} \\
\text{Im}_{3Ł} \text{ wc } \mathcal{P}_9 & = & \langle \{ab_2\}, \{e, l\} \rangle \ \rightsquigarrow\ \text{Im}_{3Ł} \text{ wc } \mathcal{P}_9(l) = \bot
\end{array}
$$

- **This logic appears to be adequate!**

Completion versus Weak Completion

- **Recall $\mathcal{P}_8$**

$$\begin{array}{lcl} \text{wc}\,\mathcal{P}_8 & = & \{l \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2),\ ab_1 \leftrightarrow \bot,\ ab_2 \leftrightarrow \bot,\ e \leftrightarrow \bot\} \\ \text{lm}_{3Ł}\,\text{wc}\,\mathcal{P}_8 & = & \langle \emptyset, \{e, ab_1, ab_2\}\rangle \ \rightsquigarrow\ \text{lm}_{3Ł}\,\text{wc}\,\mathcal{P}_8(l) = \text{U} \\ \text{c}\,\mathcal{P}_8 & = & \{l \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2),\ ab_1 \leftrightarrow \bot,\ ab_2 \leftrightarrow \bot,\ e \leftrightarrow \bot, \\ & & \ t \leftrightarrow \bot\} \\ \text{lm}_{3Ł}\,\text{c}\,\mathcal{P}_8 & = & \langle \emptyset, \{e, t, l, ab_1, ab_2\}\rangle \ \rightsquigarrow\ \text{lm}_{3Ł}\,\text{c}\,\mathcal{P}_8(l) = \bot \end{array}$$

- **A logic using completion instead of weak completion is not adequate.**

Computing the Least Models of Weakly Completed Programs

- How can we compute the least models of weakly completed programs?
- A first candidate:
  Fitting's immediate consequence operator $\Phi_{F,\mathcal{P}}(I) = \langle J^\top, J^\bot \rangle$, where

$$\begin{array}{lcl} J^\top & = & \{A \mid \text{there exists } A \leftarrow Body \in \mathcal{P} \text{ with } I(Body) = \top\} \text{ and} \\ J^\bot & = & \{A \mid \text{for all } A \leftarrow Body \in \mathcal{P} \text{ we find } I(Body) = \bot\}. \end{array}$$

- Uses F-semantics.
- Some well-known results mostly due to Fitting 1985:
  1. $\Phi_{F,\mathcal{P}}$ is monotone on $(\mathcal{I}, \subseteq)$.
  2. $\Phi_{F,\mathcal{P}}$ is continuous
     and, hence, admits a least fixed point denoted by lfp $\Phi_{F,\mathcal{P}}$.
  3. lfp $\Phi_{F,\mathcal{P}}$ can be computed by iterating $\Phi_{F,\mathcal{P}}$ on $\langle \emptyset, \emptyset \rangle$.
  4. The least F-model of c $\mathcal{P}$ is the least fixed point of $\Phi_{F,\mathcal{P}}$.
- Inadequate for human reasoning.

The Stenning and van Lambalgen Operator

- Stenning and van Lambalgen's operator $\Phi_{\text{SvL},\mathcal{P}}(I) = \langle J^\top, J^\bot \rangle$, where

$$
\begin{array}{lcl}
J^\top & = & \{A \mid \text{there exists } A \leftarrow Body \in \mathcal{P} \text{ with } I(Body) = \top\} \text{ and} \\
J^\bot & = & \{A \mid \text{there exists } A \leftarrow Body \in \mathcal{P} \text{ and} \\
 & & \qquad \text{for all } A \leftarrow Body \in \mathcal{P} \text{ we find } I(Body) = \bot\}.
\end{array}
$$

- Theorem 6
  1. $\Phi_{\text{SvL},\mathcal{P}}$ is monotone on $(\mathcal{I}, \subseteq)$.
  2. $\Phi_{\text{SvL},\mathcal{P}}$ is continuous
     and, hence, admits a least fixed point denoted by lfp $\Phi_{\text{SvL},\mathcal{P}}$.
  3. lfp $\Phi_{\text{SvL},\mathcal{P}}$ can be computed by iterating $\Phi_{\text{SvL},\mathcal{P}}$ on $\langle \emptyset, \emptyset \rangle$.
  4. $\text{lm}_{3Ł}\, \text{wc}\, \mathcal{P} = \text{lfp}\, \Phi_{\text{SvL},\mathcal{P}}$.

Computing Least Fixed Points

- Recall some of our examples

$$\begin{aligned}
\mathcal{P}_4 &= \{l \leftarrow e \wedge \neg ab,\ ab \leftarrow \bot,\ e \leftarrow \top\} \\
\text{wc}\,\mathcal{P}_4 &= \{l \leftrightarrow e \wedge \neg ab,\ ab \leftrightarrow \bot,\ e \leftrightarrow \top\} \\
\Phi_{\text{SvL},\mathcal{P}_4}(\langle \emptyset, \emptyset \rangle) &= \langle \{e\}, \{ab\} \rangle \\
\Phi_{\text{SvL},\mathcal{P}_4}(\langle \{e\}, \{ab\} \rangle) &= \langle \{e, l\}, \{ab\} \rangle \\
&= \text{lfp}\ \Phi_{\text{SvL},\mathcal{P}_4} \\
&= \text{lm}_{3Ł}\ \text{wc}\,\mathcal{P}_4
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_9 &= \{l \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \neg o, \\
&\quad\ \ l \leftarrow o \wedge \neg ab_2,\ ab_2 \leftarrow \neg e,\ e \leftarrow \bot\} \\
\text{wc}\,\mathcal{P}_9 &= \{l \leftrightarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_2), \\
&\quad\ \ ab_1 \leftrightarrow \neg o,\ ab_2 \leftrightarrow \neg e,\ e \leftrightarrow \bot\} \\
\Phi_{\text{SvL},\mathcal{P}_9}(\langle \emptyset, \emptyset \rangle) &= \langle \emptyset, \{e\} \rangle \\
\Phi_{\text{SvL},\mathcal{P}_9}(\langle \emptyset, \{e\} \rangle) &= \langle \{ab_2\}, \{e\} \rangle \\
\Phi_{\text{SvL},\mathcal{P}_9}(\langle \{ab_2\}, \{e\} \rangle) &= \langle \{ab_2\}, \{e, l\} \rangle \\
&= \text{lfp}\ \Phi_{\text{SvL},\mathcal{P}_9} \\
&= \text{lm}_{3Ł}\ \text{wc}\,\mathcal{P}_9
\end{aligned}$$

Summary

- Under Łukasiewicz semantics we obtain

	Byrne 1989	Program	$\text{Im}_{3Ł}$ wc $\mathcal{P}_i(I)$
Modus Ponens	l (96%)	$\mathcal{P}_4$	$\top$
Alternative Arguments	l (96%)	$\mathcal{P}_5$	$\top$
Additional Arguments	l (38%)	$\mathcal{P}_6$	U
Modus Ponens and DA	$\neg l$ (46%)	$\mathcal{P}_7$	$\bot$
Alternative Arguments and DA	$\neg l$ (4%)	$\mathcal{P}_8$	U
Additional Arguments and DA	$\neg l$ (63%)	$\mathcal{P}_9$	$\bot$

- The approach appears to be adequate.
- Fitting semantics or completion is inadequate.

Contraction Mappings

- **Do we have to initialize the computation of lfp $\mathcal{P}$ with the empty interpretation?**
- **Banach's Contraction Mapping Theorem**
  A contraction mapping f on a complete metric space has a unique fixed point; the sequence $x,\ f(x),\ f(f(x)),\ \ldots$ converges to this fixed point, where x is an arbitrary element from the metric space.
- Let $\mathcal{P}$ be a program. A **level mapping** is a mapping l from the set of atoms to $\mathbb{N}$. It is extended to negative atoms by defining $\mathrm{l}(\neg A) = \mathrm{l}(A)$ for each atom A.
- Let $\mathcal{I}$ be the set of all interpretations and $I, J \in \mathcal{I}$.

$$\mathrm{d_l}(I, J) = \begin{cases} \frac{1}{2^n} & \text{if } I \neq J \text{ and } I(A) = J(A) \neq \mathrm{U} \text{ for all } A \text{ with } \mathrm{l}(A) < n \text{ and} \\ & I(A) \neq J(A) \text{ or } I(A) = J(A) = \mathrm{U} \text{ for some } A \text{ with } \mathrm{l}(A) = n, \\ 0 & \text{otherwise.} \end{cases}$$

- **Proposition 7** (Kencana Ramli 2009) $(\mathcal{I}, \mathrm{d_l})$ is a complete metric space.

Contraction Properties of the Semantic Operators

- Fitting 1985 If $\mathcal{P}$ is an acceptable program then $\Phi_{F,\mathcal{P}}$ is a contraction.
  - This does not hold for $\Phi_{SvL,\mathcal{P}}$.
  - Consider $\mathcal{P} = \{r \wedge q \rightarrow p,\ r \wedge p \rightarrow q\}$.
  - $\mathcal{P}$ is acceptable.
  - Both, $\langle\emptyset, \emptyset\rangle$ and $\langle\emptyset, \{p, q\}\rangle$, are fixed points of $\Phi_{SvL,\mathcal{P}}$.
  - By Banach's contraction mapping theorem $\Phi_{SvL,\mathcal{P}}$ is not a contraction.
- Theorem 8 If $\mathcal{P}$ is an acyclic program then $\Phi_{SvL,\mathcal{P}}$ is a contraction.
  - If $\mathcal{P}$ is acyclic then we find a level mapping l such that for each $L_1 \wedge \ldots \wedge L_n \rightarrow A \in \mathcal{P}$ we have $l(L_i) < l(A)$.
  - $d_l(\Phi_{SvL,\mathcal{P}}(I), \Phi_{SvL,\mathcal{P}}(J)) \leq \frac{1}{2} d_l(I, J)$.
- Corollary 9 If $\mathcal{P}$ is an acyclic program then $\Phi_{SvL,\mathcal{P}}$ has a unique fixed point which can be reached by iterating $\Phi_{SvL,\mathcal{P}}$ starting from some interpretation.

Positive versus Negative Information

- Consider $\mathcal{P} = \{p \leftarrow \top, p \leftarrow \bot\}$.
- $\text{lm}_{3Ł}\,\text{wc}\,\mathcal{P} = \langle\{p\}, \emptyset\rangle$.
- Positive information dominates negative information.
- However, we may take integrity constraints into account, i.e. expressions of the form $L_1 \wedge \ldots \wedge L_n \rightarrow \bot$.

The Suppression Task – Affirmation of the Consequent (AC)

- Byrne: Suppressing Valid Inferences with Conditionals.
  Cognition 31, 61-83: 1989

- If she has an essay to write then she will study late in the library.
  She will study late in the library.
  - 53% of subjects conlude that she has an essay to write.

- If she has an essay to write then she will study late in the library.
  She will study late in the library.
  If she has a textbook to read she will study late in the library.
  - 16% of subjects conlude that she has an essay to write.

- If she has an essay to write then she will study late in the library.
  She will study late in the library.
  If the library stays open she will study late in the library.
  - 55% of subjects conlude that she has an essay to write.

The Suppression Task – Modus Tollens (MT)

- Byrne: Suppressing Valid Inferences with Conditionals.
  Cognition 31, 61-83: 1989
- If she has an essay to write then she will study late in the library.
  She will not study late in the library.
  - 69% of subjects conlude that she does not have an essay to write.
- If she has an essay to write then she will study late in the library.
  She will not study late in the library.
  If she has a textbook to read she will study late in the library.
  - 69% of subjects conlude that she does not have an essay to write.
- If she has an essay to write then she will study late in the library.
  She will not study late in the library.
  If the library stays open she will study late in the library.
  - 44% of subjects conlude that she does not have an essay to write.

Abductive Frameworks and Observations

- Let $\mathcal{K} \subseteq \mathcal{L}$ be a set of formulas called the knowledge base, $\mathcal{A} \subseteq \mathcal{L}$ be a set of formulas called abducibles, and $\models \subseteq 2^{\mathcal{L}} \times \mathcal{L}$ a logical consequence relation. $\langle \mathcal{K}, \mathcal{A}, \models \rangle$ is called abductive framework.
- An observation $\mathcal{O}$ is a subset of the language $\mathcal{L}$.
- Here
  - $\mathcal{K}$ is a logic program $\mathcal{P}$ (with negative facts).
  - $\mathcal{L}$ is the language underlying $\mathcal{P}$.
  - $\mathcal{R}_{\mathcal{P}}^{D} = \{A \in \mathcal{R}_{\mathcal{P}} \mid A \leftarrow Body \in \mathcal{P}\}$ is the set of defined predicates in $\mathcal{P}$.
  - $\mathcal{R}_{\mathcal{P}}^{U} = \mathcal{R}_{\mathcal{P}} \setminus \mathcal{R}_{\mathcal{P}}^{D}$ is the set of undefined predicates in $\mathcal{P}$.
  - $\mathcal{A}$ is the set $\{A \leftarrow \top \mid A \in \mathcal{R}_{\mathcal{P}}^{U}\} \cup \{A \leftarrow \bot \mid A \in \mathcal{R}_{\mathcal{P}}^{U}\}$.
  - $\models$ is $\models_{3Ł}^{lm\ wc}$, where $\mathcal{P} \models_{3Ł}^{lm\ wc} F$ iff $\text{lm}_{3Ł}\, \text{wc}\, \mathcal{P}(F) = \top$.
  - Observations are usually sets containing a single literal, in which case we simply write $\mathcal{O} = L$ instead of $\mathcal{O} = \{L\}$.

Explanations

- Let $\mathcal{L}$ be a language.
- Let $\langle \mathcal{K}, \mathcal{A}, \models \rangle$ be an abductive framework and $\mathcal{O}$ an observation.
- $\mathcal{O}$ is explained by $\mathcal{E}$ (or $\mathcal{E}$ is an explanation for $\mathcal{O}$) iff
  - $\mathcal{E} \subseteq \mathcal{A}$,
  - $\mathcal{K} \cup \mathcal{E}$ is satisfiable,
  - $\mathcal{K} \cup \mathcal{E} \models L$ for each $L \in \mathcal{O}$.
- An explanation $\mathcal{E}$ for $\mathcal{O}$ is said to be a minimal iff there is no explanation $\mathcal{E}' \subset \mathcal{E}$ for $\mathcal{O}$.

Sceptical and Credulous Reasoning

- Let $\langle \mathcal{P}, \mathcal{A}, \models_{3Ł}^{lm\ wc} \rangle$ be an abductive framework, where
  - $\mathcal{P}$ is a logic program and
  - $\mathcal{A} = \{A \leftarrow \top \mid A \in \mathcal{R}_{\mathcal{P}}^{U}\} \cup \{A \leftarrow \bot \mid A \in \mathcal{R}_{\mathcal{P}}^{U}\}$ the set of abducibles
- Let $\mathcal{O}$ be an observation and F a formula in the language underlying $\mathcal{P}$.
- F follows sceptically by abduction from $\mathcal{P}$ and $\mathcal{O}$ (in symbols $\mathcal{P}, \mathcal{O} \models_{A}^{s} F$) iff $\mathcal{O}$ can be explained and for all minimal explanations $\mathcal{E}$ we find $\mathcal{P} \cup \mathcal{E} \models_{3Ł}^{lm\ wc} F$.
- F follows credulously by abduction from $\mathcal{P}$ and $\mathcal{O}$ (in symbols $\mathcal{P}, \mathcal{O} \models_{A}^{c} F$) iff there exists a minimal explanation $\mathcal{E}$ such that $\mathcal{P} \cup \mathcal{E} \models_{3Ł}^{lm\ wc} F$.

The Suppression Task – Modus Ponens and AC

- If she has an essay to write then she will study late in the library. She will study late in the library.
  - 53% of subjects conlude that she has an essay to write.
- We obtain

$$\begin{array}{rcl} \mathcal{P}_{10} & = & \{l \leftarrow e \wedge \neg ab, ab \leftarrow \bot\} \\ \mathcal{A} & = & \{e \leftarrow \top, e \leftarrow \bot\} \\ \mathcal{O} & = & l \end{array}$$

- Thus

$$\begin{array}{rcl} \mathrm{lm}_{3Ł}\, \mathrm{wc}\, \mathcal{P}_{10} & = & \langle \emptyset, \{ab\} \rangle \\ \mathrm{lm}_{3Ł}\, \mathrm{wc}\, (\mathcal{P}_{10} \cup \{e \leftarrow \top\}) & = & \langle \{e, l\}, \{ab\} \rangle \\ \mathrm{lm}_{3Ł}\, \mathrm{wc}\, (\mathcal{P}_{10} \cup \{e \leftarrow \bot\}) & = & \langle \emptyset, \{e, l, ab\} \rangle \end{array}$$

- Hence, $\{e \leftarrow \top\}$ is the only minimal explanation and $\mathcal{P}_{10}, \mathcal{O} \models^s_{\mathcal{A}} e$.

The Suppression Task – Alternative Arguments and AC

- If she has an essay to write then she will study late in the library.
  She will study late in the library.
  If she has a textbook to read she will study late in the library.
  - 16% of subjects conlude that she has an essay to write.
- We obtain

$$\begin{array}{rcl} \mathcal{P}_{11} & = & \{l \leftarrow e \wedge \neg ab_1, ab_1 \leftarrow \bot, l \leftarrow t \wedge \neg ab_2, ab_2 \leftarrow \bot\} \\ \mathcal{A} & = & \{e \leftarrow \top, e \leftarrow \bot, t \leftarrow \top, e \leftarrow \bot\} \\ \mathcal{O} & = & l \end{array}$$

- Thus

$$\begin{array}{rcl} \mathrm{Im}_{3Ł}\ \mathrm{wc}\ \mathcal{P}_{11} & = & \langle \emptyset, \{ab_1, ab_2\}\rangle \\ \mathrm{Im}_{3Ł}\ \mathrm{wc}\ (\mathcal{P}_{11} \cup \{e \leftarrow \top\}) & = & \langle \{e, l\}, \{ab_1, ab_2\}\rangle \\ \mathrm{Im}_{3Ł}\ \mathrm{wc}\ (\mathcal{P}_{11} \cup \{e \leftarrow \bot\}) & = & \langle \emptyset, \{e, ab_1, ab_2\}\rangle \\ \mathrm{Im}_{3Ł}\ \mathrm{wc}\ (\mathcal{P}_{11} \cup \{t \leftarrow \top\}) & = & \langle \{t, l\}, \{ab_1, ab_2\}\rangle \\ \mathrm{Im}_{3Ł}\ \mathrm{wc}\ (\mathcal{P}_{11} \cup \{t \leftarrow \bot\}) & = & \langle \emptyset, \{t, ab_1, ab_2\}\rangle \end{array}$$

- Hence, $\{e \leftarrow \top\}$ and $\{t \leftarrow \top\}$ are minimal explanations and $\mathcal{P}_{11}, \mathcal{O} \not\models^{s}_{\mathcal{A}} e$.

The Suppression Task – Additional Arguments and AC

- If she has an essay to write then she will study late in the library.
  She will study late in the library.
  If the library stays open she will study late in the library.
  - 55% of subjects conlude that she has an essay to write.
- We obtain

$$\begin{array}{rcl} \mathcal{P}_{12} & = & \{l \leftarrow e \wedge \neg ab_1, ab_1 \leftarrow \neg o, l \leftarrow o \wedge \neg ab_2, ab_2 \leftarrow \neg e\} \\ \mathcal{A} & = & \{e \leftarrow \top, e \leftarrow \bot, o \leftarrow \top, o \leftarrow \bot\} \\ \mathcal{O} & = & l \end{array}$$

- Thus

$$\begin{array}{rcl} \mathrm{lm}_{3Ł}\, \mathrm{wc}\, \mathcal{P}_{12} & = & \langle \emptyset, \emptyset \rangle \\ \mathrm{lm}_{3Ł}\, \mathrm{wc}\, (\mathcal{P}_{12} \cup \{e \leftarrow \top\}) & = & \langle \{e\}, \{ab_2\} \rangle \\ \mathrm{lm}_{3Ł}\, \mathrm{wc}\, (\mathcal{P}_{12} \cup \{e \leftarrow \bot\}) & = & \langle \{ab_2\}, \{e, l\} \rangle \\ \mathrm{lm}_{3Ł}\, \mathrm{wc}\, (\mathcal{P}_{12} \cup \{o \leftarrow \top\}) & = & \langle \{t\}, \{ab_1\} \rangle \\ \mathrm{lm}_{3Ł}\, \mathrm{wc}\, (\mathcal{P}_{12} \cup \{o \leftarrow \bot\}) & = & \langle \{ab_1\}, \{o, l\} \rangle \\ \mathrm{lm}_{3Ł}\, \mathrm{wc}\, (\mathcal{P}_{12} \cup \{e \leftarrow \top, o \leftarrow \top\}) & = & \langle \{t, e, l\}, \{ab_1, ab_2\} \rangle \end{array}$$

- $\{e \leftarrow \top, o \leftarrow \top\}$ is the only minimal explanation and $\mathcal{P}_{12}, \mathcal{O} \models^s_{\mathcal{A}} e$.

The Suppression Task – Modus Tollens (MT)

- If she has an essay to write then she will study late in the library.
  She will not study late in the library.
  - 69% of subjects conlude that she does not have an essay to write.
- We obtain

$$\begin{array}{rcl} \mathcal{P}_{10} & = & \{l \leftarrow e \wedge \neg ab, ab \leftarrow \bot\} \\ \mathcal{A} & = & \{e \leftarrow \top, e \leftarrow \bot\} \\ \mathcal{O} & = & \neg l \end{array}$$

- Thus

$$\begin{array}{rcl} \text{lm}_{3Ł}\, \text{wc}\, \mathcal{P}_{10} & = & \langle \emptyset, \{ab\} \rangle \\ \text{lm}_{3Ł}\, \text{wc}\, (\mathcal{P}_{10} \cup \{e \leftarrow \top\}) & = & \langle \{e, l\}, \{ab\} \rangle \\ \text{lm}_{3Ł}\, \text{wc}\, (\mathcal{P}_{10} \cup \{e \leftarrow \bot\}) & = & \langle \emptyset, \{e, l, ab\} \rangle \end{array}$$

- Hence, $\{e \leftarrow \bot\}$ is the only minimal explanation and $\mathcal{P}_{10}, \mathcal{O} \models_{\mathcal{A}}^{s} \neg e$.

The Suppression Task – Alternative Arguments and MT

- If she has an essay to write then she will study late in the library.
  She will not study late in the library.
  If she has a textbook to read she will study late in the library.
  - 69% of subjects conlude that she does not have an essay to write.
- We obtain

$$\begin{array}{rcl} \mathcal{P}_{11} & = & \{l \leftarrow e \wedge \neg ab_1, ab_1 \leftarrow \bot, l \leftarrow t \wedge \neg ab_2, ab_2 \leftarrow \bot\} \\ \mathcal{A} & = & \{e \leftarrow \top, e \leftarrow \bot, t \leftarrow \top, t \leftarrow \bot\} \\ \mathcal{O} & = & \neg l \end{array}$$

- Thus

$$\begin{array}{rcl} \text{lm}_{3Ł}\,\text{wc}\,\mathcal{P}_{11} & = & \langle \emptyset, \{ab_1, ab_2\}\rangle \\ \text{lm}_{3Ł}\,\text{wc}\,(\mathcal{P}_{11} \cup \{e \leftarrow \top\}) & = & \langle \{e, l\}, \{ab_1, ab_2\}\rangle \\ \text{lm}_{3Ł}\,\text{wc}\,(\mathcal{P}_{11} \cup \{e \leftarrow \bot\}) & = & \langle \emptyset, \{e, ab_1, ab_2\}\rangle \\ \text{lm}_{3Ł}\,\text{wc}\,(\mathcal{P}_{11} \cup \{t \leftarrow \top\}) & = & \langle \{t, l\}, \{ab_1, ab_2\}\rangle \\ \text{lm}_{3Ł}\,\text{wc}\,(\mathcal{P}_{11} \cup \{t \leftarrow \bot\}) & = & \langle \emptyset, \{t, ab_1, ab_2\}\rangle \\ \text{lm}_{3Ł}\,\text{wc}\,(\mathcal{P}_{11} \cup \{e \leftarrow \bot, t \leftarrow \bot\}) & = & \langle \emptyset, \{e, l, t, ab_1, ab_2\}\rangle \end{array}$$

- $\{e \leftarrow \bot, t \leftarrow \bot\}$ is the only minimal explanation and $\mathcal{P}_{11}, \mathcal{O} \models^s_{\mathcal{A}} \neg e$.

The Suppression Task – Additional Arguments and MT

- If she has an essay to write then she will study late in the library.
  She will not study late in the library.
  If the library stays open she will study late in the library.
  - 44% of subjects conlude that she does not have an essay to write.
- We obtain

$$\begin{array}{rcl} \mathcal{P}_{12} & = & \{l \leftarrow e \wedge \neg ab_1, ab_1 \leftarrow \neg o, l \leftarrow o \wedge \neg ab_2, ab_2 \leftarrow \neg e\} \\ \mathcal{A} & = & \{e \leftarrow \top, e \leftarrow \bot, o \leftarrow \top, o \leftarrow \bot\} \\ \mathcal{O} & = & \neg l \end{array}$$

- Thus

$$\begin{array}{rcl} \mathrm{Im}_{3Ł}\,\mathrm{wc}\,\mathcal{P}_{12} & = & \langle \emptyset, \emptyset \rangle \\ \mathrm{Im}_{3Ł}\,\mathrm{wc}\,(\mathcal{P}_{12} \cup \{e \leftarrow \top\}) & = & \langle \{e\}, \{ab_2\} \rangle \\ \mathrm{Im}_{3Ł}\,\mathrm{wc}\,(\mathcal{P}_{12} \cup \{e \leftarrow \bot\}) & = & \langle \{ab_2\}, \{e, l\} \rangle \\ \mathrm{Im}_{3Ł}\,\mathrm{wc}\,(\mathcal{P}_{12} \cup \{o \leftarrow \top\}) & = & \langle \{t\}, \{ab_1\} \rangle \\ \mathrm{Im}_{3Ł}\,\mathrm{wc}\,(\mathcal{P}_{12} \cup \{o \leftarrow \bot\}) & = & \langle \{ab_1\}, \{o, l\} \rangle \end{array}$$

- Hence, $\{e \leftarrow \bot\}$ and $\{o \leftarrow \bot\}$ are minimal explanations and $\mathcal{P}_{12}, \mathcal{O} \not\models^s_{\mathcal{A}} \neg e$.

Weak Completion is Needed

- Reconsider the case modus ponens with positive observation, i.e.
  - $\mathcal{P}_{10} = \{l \leftarrow e \wedge \neg ab, ab \leftarrow \bot\}$,
  - $\mathcal{A} = \{e \leftarrow \top, e \leftarrow \bot\}$, and
  - $\mathcal{O} = l$.
- Now consider $\langle \mathcal{P}_{10}, \mathcal{A}, \models_{3Ł} \rangle$ instead of $\langle \mathcal{P}_{10}, \mathcal{A}, \models_{3Ł}^{lm\ wc} \rangle$
  - $\mathcal{P}_{10} \not\models_{3Ł} l$
  - $\mathcal{P}_{10} \cup \{e \leftarrow \top\} \not\models_{3Ł} l$ (because ab can be mapped to $\top$).
  - $\mathcal{P}_{10} \cup \{e \leftarrow \bot\} \not\models_{3Ł} l$
  - $\mathcal{P}_{10} \cup \mathcal{A} \not\models_{3Ł} l$

  Hence, the observation can not be explained at all (in contrast to Byrne 1989).

Completion is Insufficient

- Reconsider the case modus ponens with positive observation, i.e.
  - $\mathcal{P}_{10} = \{l \leftarrow e \wedge \neg ab, ab \leftarrow \bot\}$,
  - $\mathcal{A} = \{e \leftarrow \top, e \leftarrow \bot\}$, and
  - $\mathcal{O} = l$.
- Now consider $\langle \mathcal{P}_{10}, \mathcal{A}, \models^{c}_{3Ł} \rangle$ instead of $\langle \mathcal{P}_{10}, \mathcal{A}, \models^{lm\ wc}_{3Ł} \rangle$, where $\mathcal{P} \models^{c}_{3Ł} F$ iff F holds in all models for c $\mathcal{P}$.
  - c $\mathcal{P}_{10} = \{l \leftrightarrow e \wedge \neg ab, ab \leftrightarrow \bot, e \leftrightarrow \bot\} \models_{3Ł} \neg l$
  - c $\mathcal{P}_{10} \models_{3Ł} \neg e$

  The observation is inconsistent with the knowledge base and, thus, cannot be explained at all (in contrast to Byrne 1989).

Explanations must be Completed as well

- Reconsider the case modus ponens with negative observation, i.e.
  - $\mathcal{P}_{10} = \{l \leftarrow e \wedge \neg ab, ab \leftarrow \bot\}$,
  - $\mathcal{A} = \{e \leftarrow \top, e \leftarrow \bot\}$, and
  - $\mathcal{O} = \neg l$.
- Now (weakly) complete only the program, but not the explanations.
  - $\text{wc}\,\mathcal{P}_{10} = \{l \leftrightarrow e \wedge \neg ab, ab \leftrightarrow \bot\}$.
  - $\text{wc}\,\mathcal{P}_{10} \not\models_{3Ł} \neg l$
  - $\text{wc}\,(\mathcal{P}_{10}) \cup \{e \leftarrow \top\} \not\models_{3Ł} \neg l$
  - $\text{wc}\,(\mathcal{P}_{10}) \cup \{e \leftarrow \bot\} \not\models_{3Ł} \neg l$ (because e can be mapped to $\top$).
  - $\text{wc}\,(\mathcal{P}_{10}) \cup \mathcal{A} \not\models_{3Ł} \neg l$

  Hence, the observation can not be explained (in contrast with Byrne 1989).

Sceptical versus Creduluos Reasoning

- Reconsider the case alternative arguments with positive observation, i.e.,
  - $\mathcal{P}_{11} = \{l \leftarrow e \wedge \neg ab_1, ab_1 \leftarrow \bot, l \leftarrow t \wedge \neg ab_2, ab_2 \leftarrow \bot\}$,
  - $\mathcal{A} = \{e \leftarrow \top, e \leftarrow \bot\}$, and
  - $\mathcal{O} = l$.
- Now consider $\langle \mathcal{P}_{11}, \mathcal{A}, \models_{3Ł}^{lm\ wc} \rangle$ and reason credulously:
  - There are two minimal explanations, viz. $\{e \leftarrow \top\}$ and $\{e \leftarrow \bot\}$.
  - Hence, $\mathcal{P}_{11}, l \not\models_{\mathcal{A}}^{s} e$, but $\mathcal{P}_{11}, l \models_{\mathcal{A}}^{c} e$.

  Creduluos reasoning is inconsistent with Byrne 1989.

Summary

- Let $\mathcal{P}_{10} = \{l \leftarrow e \wedge \neg ab, ab \leftarrow \bot\}$
  $\mathcal{P}_{11} = \{l \leftarrow e \wedge \neg ab_1, ab_1 \leftarrow \bot, l \leftarrow t \wedge \neg ab_2, ab_2 \leftarrow \bot\}$
  $\mathcal{P}_{12} = \{l \leftarrow e \wedge \neg ab_1, ab_1 \leftarrow \neg o, l \leftarrow o \wedge \neg ab_2, ab_2 \leftarrow \neg e\}$
- We obtain

	Byrne 1989
$\mathcal{P}_{10}, l \models^s_A e$	$e(53\%)$
$\mathcal{P}_{11}, l \not\models^s_A e$	$e(16\%)$
$\mathcal{P}_{12}, l \models^s_A e$	$e(55\%)$
$\mathcal{P}_{10}, \neg l \models^s_A \neg e$	$\neg e(69\%)$
$\mathcal{P}_{11}, \neg l \models^s_A \neg e$	$\neg e(69\%)$
$\mathcal{P}_{12}, \neg l \not\models^s_A e$	$\neg e(44\%)$

Summary

- Logic appears to be adequate for the suppression task if
  - weak completion,
  - Łukasiewicz semantics,
  - the Stenning and van Lambalgen semantic operator, and
  - abduction are used.
- Human Reasoning is modeled by
  - reasoning towards an appropriate logic program $\mathcal{P}$ and, thereafter,
  - reasoning with respect to the least Ł-model of the weak completion of $\mathcal{P}$.
- This approach matches data from studies in human reasoning.
- There is a connectionist encoding.

Discussion

- Stenning, van Lambalgen 2008 propose spreading-activation networks like KBANN (Towell, Shavlik 1993) with two units for each propositional letter and an inhibitory link between them.
- Logical threshold units can be replaced by bipolar sigmoidal ones following d'Avila Garcez, Zaverucha, Carbalho 1997.
  - Networks can be trained by backpropagation,
  - but backpropagation is not neurally plausible.

Some Open Problems (1)

- **Negation**
  - How is negation treated in human reasoning?
- **Errors**
  - How can frequently made errors be explained in the proposed approach?
- **Łukasiezicz logic**
  - In a Łukasiezicz logic the semantic deduction theorem does not hold.
  - Is this adequate with respect to human reasoning?
- **Completion**
  - Under which conditions is human reasoning adequately modeled by completion and/or weak completion?

Some Open Problems (2)

- **Contractions**
  - Do humans exhibit a behavior which can be adequately modeled by contractional semantic operators?
  - Can we generate appropriate level mappings by studying the behavior of humans?
- **Explanations**
  - Do humans consider minimal explanations?
  - In which order are (minimal) explanations generated by humans if there are several?
  - Does attention play a role in the selection of (minimal) explanations?
- **Stable coalitions**
  - Do stable coalitions occur in human reasoning?
  - How are they deactivated?

Some Open Problems (3)

- **Reasoning**
  - **Do humans reason sceptically or credulously?**
  - **How does a connectionist realization of sceptical reasoning looks like?**
- **Theory revision**
  - **How is theory revision modeled in human reasoning?**
- **Relation to other semantics**
  - **What is the relation between the proposed approach and well-founded and/or stable and/or projection semantics?**