Communication Issues in Collective Decision Making

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Overview of this course

In this course I discuss communication issues related to different collective decision making problems.

1. This morning I introduce three different collective decision making problems, and show in particular why they raise interesting computational problems. (Part I)
2. After coffee break, I investigate in particular communication issues related to these problems. (Part II).

Overview of this course

Collective decision-making:

- a set of agents $\mathcal{N}$, a set of "options" $\mathcal{O}$
- agents have (potentially conflicting) preferences about the options
- have to agree on a decision (choice of an option)

1. voting ($\mathcal{O}$ = set of candidates)
2. two-sided matching ($\mathcal{O}$ = set of matchings, preferences about agents from the other side)
3. resource allocation ($\mathcal{O}$ = set of allocations, preferences about bundle of resources they hold)

Note: Where is computational logic here?

In this talk I will signal a "Logic Alert" with this symbol: 

Outline of the Talk

Motivation
Quest for the "best" voting system

Figure: Referendum on Alternative Vote (UK, 2011)

Motivation
Manipulating the system: Gerrymandering

Figure: The salamander of Elbridge Gerry (1812)

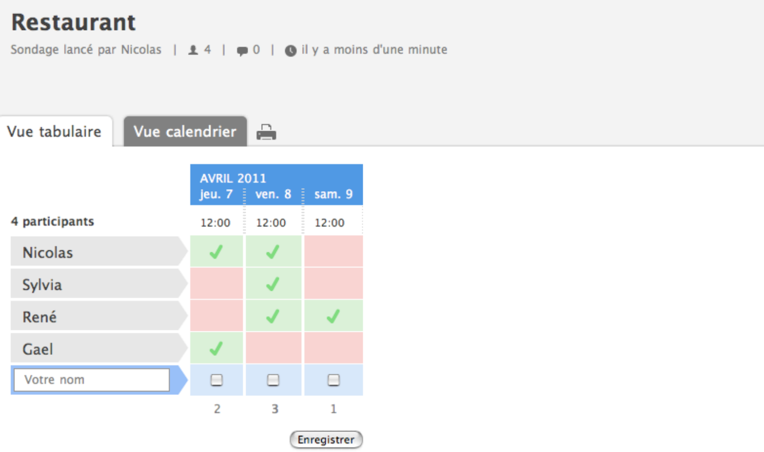

Figure: Choice of a restaurant

Motivation
Meta-search engines

Figure: Aggregating search results

Voting: Definitions

1. a finite set of voters $\mathcal{A} = \{1, ..., n\}$;
2. a finite set of candidates (alternatives) $\mathcal{O}$;
3. a profile = a preference relation (= linear order) on $\mathcal{O}$ for each voter

$$P = (V_1, \ldots, V_n) = (\succ_1, \ldots, \succ_n)$$

   V_i (or $\succ_i$) = vote expressed by voter i.
4. $\mathcal{P}^n$ set of all profiles.

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- Voting rule $F : \mathcal{P}^n \to \mathcal{O}$
  $F(V_1, \ldots, V_n)$ = socially preferred (elected) candidate
- Voting correspondence $C : \mathcal{P}^n \to 2^{\mathcal{O}} \setminus \{\emptyset\}$
  $C(V_1, \ldots, V_n)$ = set of socially preferred candidates.
- Social welfare function $H : \mathcal{P}^n \to \mathcal{P}$
  $H(V_1, \ldots, V_n)$ = social preference relation ($\succ_P$)

Note: Rules can be obtained from correspondences by tie-breaking (usually by using a predefined priority order on candidates).

Positional scoring rules

- n voters, p candidates
- fixed list of p integers $s_1 \geq \ldots \geq s_p$
- voter i ranks candidate x in position $j \Rightarrow score_i(x) = s_j$
- winner: candidate maximizing $s(x) = \sum_{i=1}^{n} score_i(x)$

Examples:

- $s_1 = 1,\ s_2 = \ldots = s_m = 0 \Rightarrow$ *plurality* ;
- $s_1 = s_2 = \ldots = s_{m-1} = 1,\ s_m = 0 \Rightarrow$ *veto* ;
- $s_1 = m-1,\ s_2 = m-2,\ \ldots s_m = 0 \Rightarrow$ *Borda*.

2 voters	1 voter	1 voter
c	a	d
b	b	a
a	d	b
d	c	c

plurality	Borda
$a \mapsto 1$	$a \mapsto 6$
$b \mapsto 0$	$b \mapsto 7$
$c \mapsto 2$	$c \mapsto 6$
$d \mapsto 1$	$d \mapsto 4$
c winner	b winner

Condorcet winner

$N(x,y) = \{i | x \succ_i y\}$ set of voters who prefer x to y.
$\#N(x,y)$ number of voters who prefer x to y.

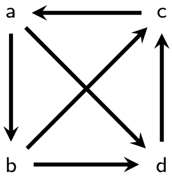
Condorcet winner

for $P = \langle \succ_1, \ldots, \succ_n \rangle$: a candidate x such that $\forall y \neq x$, $\#N(x,y) > \frac{n}{2}$
(a candidate who beats any other candidate by a majority of votes).

a	d	c
b	b	a
d	c	b
c	a	d

2 voters out of 3: $a \succ b$
2 voters out of 3: $c \succ a$
2 voters out of 3: $a \succ d$
2 voters out of 3: $b \succ c$
2 voters out of 3: $b \succ d$
2 voters out of 3: $d \succ c$

Majority graph



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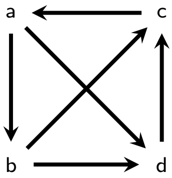

$\hookrightarrow$ No Condorcet winner

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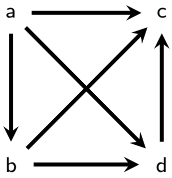

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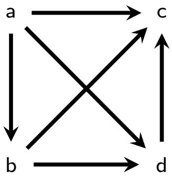

$\hookrightarrow$ a is the Condorcet winner

- Consistency with Condorcet: the voting rule should elect the Condorcet winner whenever there is one.
- Example: Copeland rule
  get 1 pt for each pairwise win, $\frac{1}{2}$ for a tie, 0 otherwise

2	1	2
a	d	c
b	b	a
d	c	b
c	a	d

4 voters out of 5: $a \succ b$
3 voters out of 5: $c \succ a$
4 voters out of 5: $a \succ d$
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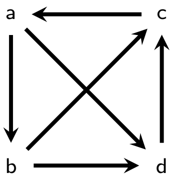
Majority graph



Condorcet-consistent rules

The Copeland rule

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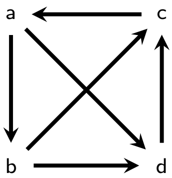

C(a) = 2
C(b) = 2
C(c) = 1
C(d) = 1

Condorcet-consistent rules

The Simpson rule

- Consistency with Condorcet: the voting rule should elect the Condorcet winner whenever there is one.
- Example: Simpson rule
  pick the candidate who minimizes the max pairwise defeat

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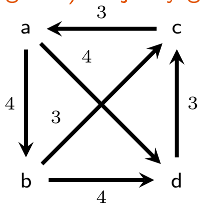
(Weighted) Majority graph



Condorcet-consistent rules

The Simpson rule

- **Consistency with Condorcet**: the voting rule should elect the Condorcet winner whenever there is one.
- Example: **Simpson rule**
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2	1	2
a	d	c
b	b	a
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4 voters out of 5: $a \succ b$
3 voters out of 5: $c \succ a$
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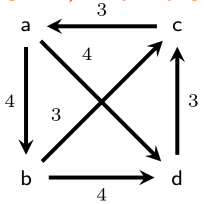
(Weighted) Majority graph



S(a) = max { 3 }
S(b) = max { 4 }
S(c) = max { 3, 3 }
S(d) = max { 4, 4 }

if there exists a candidate c ranked first by a majority of votes
then c wins
else Repeat
let d be the candidate ranked first by the fewest voters;
eliminate d from all ballots
{votes for d transferred to the next best remaining candidate};
Until there exists a candidate c ranked first by a majority of votes

3	4	3	2
a	b	c	d
d	d	d	c
b	a	a	b
c	c	b	a

Sequential Rules

Simple transferable vote (STV)

if there exists a candidate c ranked first by a majority of votes
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a	b	c	d
d	d	d	c
b	a	a	b
c	c	b	a

3	4	3	2
a	b	c	c
b	a	a	b
c	c	b	a

7	5
b	c
c	b

Winner: b

- with only 3 candidates, STV coincides with plurality with runoff.
- system used in Australia, Ireland

Approval Voting

Here the input provided by the voters is different.

- a profile = a subset of candidates $A_i \subseteq \mathcal{X}$ for each voter

$$P = (A_1, \ldots, A_n)$$

- $S_P(x)$ = number of voters i such that $x \in A_i$.
- winner = candidate maximizing S_P.

Paradoxes

The study of voting rules unveiled many "paradoxes"...

Example (Saari, 1995)

6	5	4
a	c	b
b	b	c
c	a	a

- Veto, Condorcet and Borda agree on the ranking $b \succ c \succ a$
  But plurality instead says $a \succ c \succ b$
- Other results show striking distinctions between rules, eg:
  No positional rule is Condorcet-consistent (Young)

An Axiomatic Approach

- Most results in (classical) social choice seek characterizations of voting rules in terms of axioms they fulfill.
- There are other ways to "rationalize" the use of certain voting rules:
  - maximum likelihood approach (there is a correct outcome, and the votes are noisy/distorded perceptions of this outcome, for a given model of noise)
  - distance-based rationalization (there is a consensus notion, and the winner is the winning candidate in the closest consensual profile, for a given notion of distance)

Elkind, Faliszewski & Slinko. *Distance Rationalization of Voting Rules*. COMSOC, 2010.

Conitzer & Sandholm. *Common Voting Rules as Maximum Likelihood Estimators*. UAI, 2005.

An Axiomatic Approach
[Arrow]

Sometimes impossibility results state that no voting rule can satisfy a given set of axioms.

- unanimity if $x \succ_i y$ for every voter i, then $x \succ_P y$
- independence of irrelevant alternative the social preference among x and y only depends on their relative relative ranking by every individual.
  $N^P(x,y) = N^{P'}(x,y)$ then $x \succ_P y \Leftrightarrow x \succ_{P'} y$
- dictatorship a voter i is a dictator if the function maps any profile to his vote, *i.e.* $H : \mathcal{P}^n \rightarrow V_i$

Theorem (Arrow, 1951)

Any social welfare function for 3 or more candidates satisfying unanimity and independence must be a dictatorship.


Logic and automated reasoning can be used to prove such results

Grandi & Endriss. *FOL Formalisation of Impossibility Theorems in Preference Aggregation*. JPL-2013.

An Axiomatic Approach
[May]

An example of a possibility result...

- anonymity does not depend on the identity of voters, *i.e.* $F(V_1, \ldots, V_n) = F(\pi(V_1), \ldots, \pi(V_n))$
- neutrality does not depend on the identity of candidates
- positive responsiveness if a candidate x is among the winners, then it should become the unique winner when some voters modify their preference and put x^* at a higher rank (without modifying the rest).

Theorem (May, 1952)

A voting correspondence for exactly 2 candidates satisfies anonymity, neutrality, and positive responsiveness iff it is the plurality rule (simple majority).

Paradoxes: the french system cannot be bugged...

Plurality with runoff fails to meet positive responsiveness...

6	5	6
a	b	c
b	a	b
c	c	a

1st round: b eliminated
2nd round: a elected (11/6)

Paradoxes: the french system cannot be bugged...

Plurality with runoff fails to meet positive responsiveness...

6	5	4	2
a	b	c	a
b	a	b	c
c	c	a	b

1st round: c eliminated
2nd round: b elected (9/8)

An Axiomatic Approach
[Gibbard-Satherwaite]

Another important notion is that of strategy-proofness.

A voting rule is strategy-proof if no voter is better-off (*i.e.* prefers the new obtained winner) misrepresenting his vote (in any profile).

- surjectivity no candidate is discarded (for any candidate x, there is a profile P such that $F(P) = x$)

Theorem (Gibbard-Satherwaite, 1952)

Any voting rule for 3 or more candidates that is surjective and strategy-proof must be dictatorship.

Early motivations for computational social choice

In all these results, no consideration for computational issues

- are some rules difficult to compute?
- how about the difficulty of manipulating the election?
- how do these rules cater in distributed environment?
- what if the number of candidates is huge?

Computing the winner

For most voting rules, the winner can be computed in polynomial time
Examples:

- positional scoring rules, approval: $O(np)$
- Copeland, Simpson, STV: $O(np^2)$

But for some voting rules it is NP-hard.

Reference papers

Faliszewski, Hemaspaandra, Hemaspaandra & Rothe. *A richer Understanding of the Complexity of Election Systems*. CoRR-2006.

Bartholdi, Tovey, & Trick. *Voting Schemes for which It Can Be Difficult to Tell Who Won the Election*. Social Choice and Welfare, 1992.

Hudry. *Median linear orders : heuristics and a branch and bound algorithm*. EJOR-1989.

Hard rules
Kemeny

Looking for rankings that are as "close" as possible to the preference profile and chooses the top-ranked candidates in these rankings.

- Kemeny distance:
  $d_K(V, V')$ = number of $(x, y) \in \mathcal{O}^2$ on which V and V' disagree

$$d_K(V, \langle V_1, \ldots, V_n \rangle) = \sum_{i=1,\ldots,n} d_K(V, V_i)$$

- Kemeny consensus = linear order $\succ_P$ such that $d_K(\succ_P, \langle V_1, \ldots, V_n \rangle)$ minimum
- Kemeny winner = candidate ranked first in a Kemeny consensus

A characterization of Kemeny With each profile P associate the pairwise comparison matrix (recall $\#N^P(x,y)$ is the number of voters who prefer x to y in P).

Now given a ranking R:

$$K(R) = \sum_{x \succ_R y} \#N(x,y)$$

- If $x \succ_R y$ then this corresponds to $\#N(x,y)$ agreements (and $\#N(y,x)$ disagreements)
- P^* is a Kemeny consensus iff $K(P^*)$ is maximum.

4 voters	3 voters	2 voters
a	b	c
b	c	a
c	a	b

Find the Kemeny winner(s).

Hard rules
Kemeny

4 voters	3 voters	2 voters
a	b	c
b	c	a
c	a	b

N	a	b	c
a	–	6	4
b	3	–	7
c	5	2	–

Kemeny scores

abc	acb	bac	bca	cab	cba
17	12	14	15	13	10

Kemeny consensus: abc; Kemeny winner: a

- this naive approach yields $O(p!p^2n)$

Hard rules
Kemeny

- early results: Kemeny is NP-hard (Orlin, 81; Bartholdi *et al.*, 89; Hudry, 89)
- deciding whether a candidate is a Kemeny winner is not even in NP, but higher up
- many works on approximation

Hard rules

Dodgson / Young

Other examples of rules difficult to compute:

Dodgson (= Lewis Carrol) rule for each candidate c, compute $D(c)$ the number of adjacent swaps required to turn it into a Condorcet winner. Pick the candidate minimizing $D(c)$.

- Deciding whether a designated candidate x is a Dodgson winner is NP-hard, not in NP, but higher up in the hierarchy. So even verifying is not easy.

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Dodgson / Young

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Young rule for each candidate c, compute $Y(c)$ the smallest number of voters that we need to remove to turn it into a Condorcet winner. Pick the candidate minimizing $Y(c)$.

- Deciding whether a designated candidate x is a Young winner is NP-hard, not in NP, but higher up in the hierarchy.

Hemaspaandra, Hemaspaandra, & Rothe. *Exact Analysis of Dodgson Elections*. J. of ACM, 1997.

Rothe, Spakowski, & Vogel. *Exact Complexity of the Winner Problem for Young Elections*. Theory Comput. Syst. 2003.

Motivation

A very simple setting:

- Let M be the set of men, and W be the set of women.
- Each agent of M has preferences over agents in W, and vice-versa.
- We want to match them in way that is stable.

Stimulated a huge amount of works:

- many applications, from labour markets to P2P networks...
- extremely well-studied, including recent 2012 Nobel prize winners (Roth and Shapley, "for the theory of stable allocations and the practice of market design")

Two-sided Matching: Definitions

- a matching assigns each element of M to W.
- a blocking pair are two agents no matched but which would *both* prefer to be matched together than with their current partner.
- a matching is stable when it has no blocking pair.

Example

$$\begin{array}{llll} m_1: & w_1 \succ w_2 \succ w_3 & w_1: & m_2 \succ m_3 \succ m_1 \\ m_2: & w_2 \succ w_3 \succ w_1 & w_2: & m_3 \succ m_1 \succ m_2 \\ m_3: & w_3 \succ w_2 \succ w_1 & w_3: & m_1 \succ m_2 \succ m_3 \end{array}$$

- Is $\{(m_1, w_2), (m_2, w_3), (m_3, w_1)\}$ stable?

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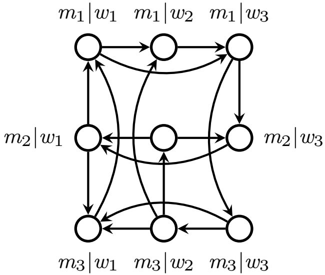
- Is $\{(m_1, w_2), (m_2, w_3), (m_3, w_1)\}$ stable? No. (m_3, w_2) is a blocking pair.
- Can you find another stable matching on this instance?

Argumentation and stable marriages

The stable marriage problem can be encoded as an argumentation framework, with $A = M \times W$, and $R \subseteq A \times A$:
(m, w_1) attacks (m, w_2) when m prefers w_1 over w_2
(m_1, w) attacks (m_2, w) when w prefers m_1 over m_2



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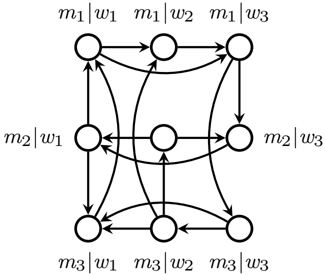

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Dung. *On the acceptability of arguments...*. AIJ-1995.

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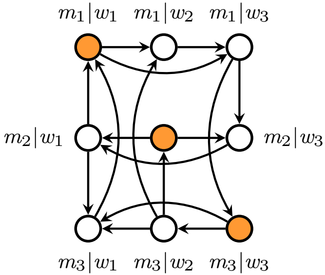

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Gale-Shapley Algorithm

The following algorithm is the "basic" Gale-Shapley algorithm

```
free all men and women
while some man m is free do
begin
  w:= first woman on m list to whom m has not yet proposed
  if w is free then
   assign m and w to be engaged
   else
    if w prefers m to her current fiance mc then
       assign (m,w) to be engaged and free mc
end;
```

- the algorithm is guaranteed to find a stable matching, in $O(n^2)$
- the choice of which man is next to propose is irrelevant
- the matching obtained is "male-optimal": all men have their preferred partner in any stable matching

Gusfield & Irving. *The stable marriage problem*. MIT Press.

Solving stable matching

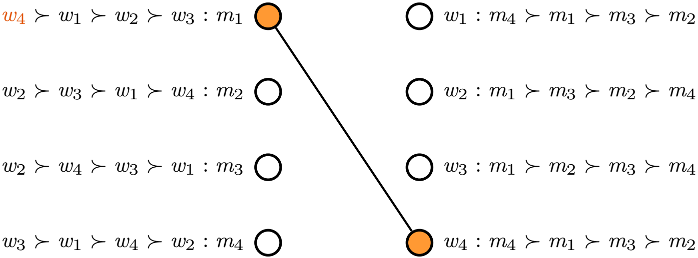
$w_4 \succ w_1 \succ w_2 \succ w_3 : m_1$ ○ ○ $w_1 : m_4 \succ m_1 \succ m_3 \succ m_2$

$w_2 \succ w_3 \succ w_1 \succ w_4 : m_2$ ○ ○ $w_2 : m_1 \succ m_3 \succ m_2 \succ m_4$

$w_2 \succ w_4 \succ w_3 \succ w_1 : m_3$ ○ ○ $w_3 : m_1 \succ m_2 \succ m_3 \succ m_4$

$w_3 \succ w_1 \succ w_4 \succ w_2 : m_4$ ○ ○ $w_4 : m_4 \succ m_1 \succ m_3 \succ m_2$

Solving stable matching



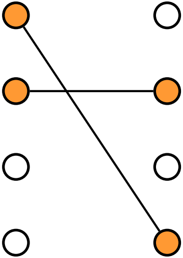
Solving stable matching

$w_4 \succ w_1 \succ w_2 \succ w_3 : m_1$

$w_2 \succ w_3 \succ w_1 \succ w_4 : m_2$

$w_2 \succ w_4 \succ w_3 \succ w_1 : m_3$

$w_3 \succ w_1 \succ w_4 \succ w_2 : m_4$



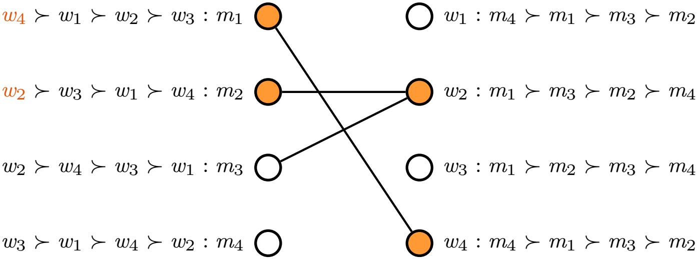
$w_1 : m_4 \succ m_1 \succ m_3 \succ m_2$

$w_2 : m_1 \succ m_3 \succ m_2 \succ m_4$

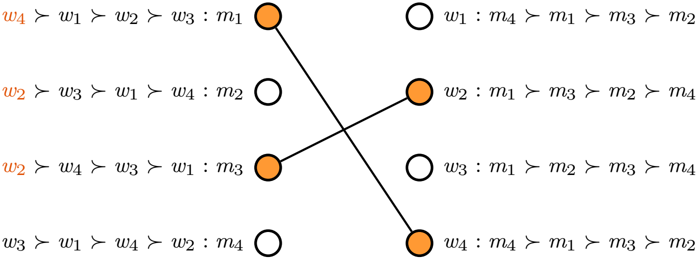
$w_3 : m_1 \succ m_2 \succ m_3 \succ m_4$

$w_4 : m_4 \succ m_1 \succ m_3 \succ m_2$

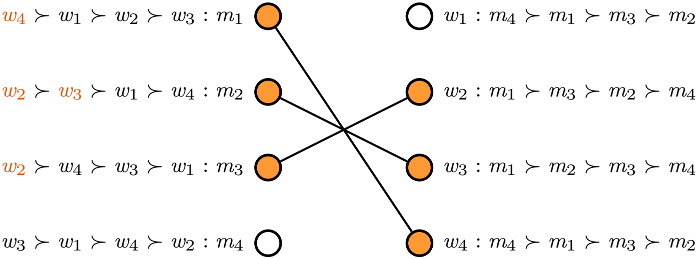
Solving stable matching



Solving stable matching



Solving stable matching



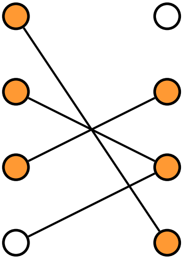
Solving stable matching

$w_4 \succ w_1 \succ w_2 \succ w_3 : m_1$

$w_2 \succ w_3 \succ w_1 \succ w_4 : m_2$

$w_2 \succ w_4 \succ w_3 \succ w_1 : m_3$

$w_3 \succ w_1 \succ w_4 \succ w_2 : m_4$



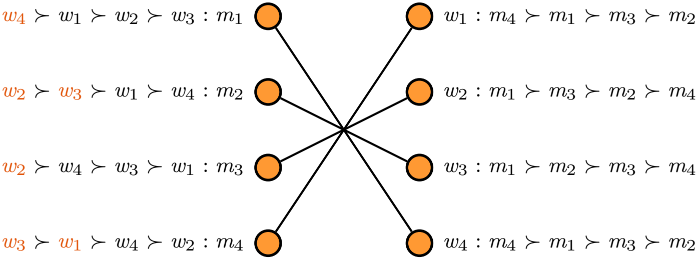
$w_1 : m_4 \succ m_1 \succ m_3 \succ m_2$

$w_2 : m_1 \succ m_3 \succ m_2 \succ m_4$

$w_3 : m_1 \succ m_2 \succ m_3 \succ m_4$

$w_4 : m_4 \succ m_1 \succ m_3 \succ m_2$

Solving stable matching



Decentralized stable matching

- the setting studied so far was centralized
- it is also possible to study a decentralized version.

Uncoordinated two-sided markets suppose the matching evolves as a consequence of self-interested agents.

```
start from a matching
while there is some blocking pair
   select a blocking pair and satisfy it
end;
```

Different choices can be made on the way you select the blocking pair to satisfy (randomly, adversary, the best possible one, etc.)
This yields different dynamics which may exhibit very different properties.

- But the existence of a path to stability is guaranteed from any matching.

Roth & Vande Vate. *Random paths to stability in two-sided matchings*. Econometrica-1990.

Outline of the Talk

Distributed Resource Allocation

- agents have to agree on a partition of goods $\mathcal{G}$.
- agents have preferences over bundles of goods they may hold $2^{\mathcal{G}}$

	u_1	u_2
$\emptyset$	0	0
$\{r_1\}$	1	3
$\{r_2\}$	3	3
$\{r_1, r_2\}$	7	8

- there is a social welfare measure to optimize, *e.g.*

$$sw(A) = \sum_{i \in \mathcal{A}} u_i(A)$$

- the problem can be tackled centrally via auctions

ASP has been used to model/solve such problems

Leite *et al.*. *Resource allocation with answer-set programming*. AAMAS-09.

We mostly discuss the decentralized version here:

- negotiation starts with an initial allocation
- agents asynchronously negotiate resources
- deals to move from one allocation to another, ie $\delta = (A, A')$
- deals can involve payments (utility transfer);
- agents accept deals on the basis of a rationality criterion, we assume myopic IR: $v_i(A') - v_i(A) > p(i)$

Contract-Based Negotiation

Different types of deals can be considered—
"natural" restrictions on the type of exchanges allowed between agents, in particular:

- 1-deals: exchange of a single resource
- bilateral deal: exchange involving two agents
- clique deal: exchange among agents in a clique of neighbours

Different assumptions on the preference structures—
"natural" restrictions/assumptions to be made on the preferences of all the agents of the system, in particular:

- monotonicity: $v_i(B_1) \leq v_i(B_2)$ when $B_1 \subseteq B_2$
- modularity: $v(S_1 \cup S_2) = v(S_1) + v(S_2) - v(S_1 \cap S_2)$

Contract-Based Negotiation

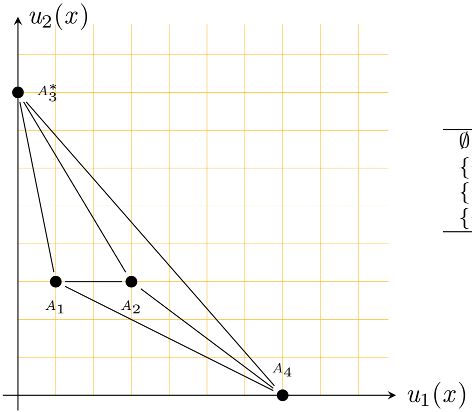
Some known results:

- a deal is IR (with money) iff it increases utilitarian social welfare (thus generates a surplus).
- allows to show that any sequence of IR deals converges to an allocation maximizing utilitarian social welfare
- however, may require very complex deals to be implemented during the negotiation (in fact, for any conceivable deal we may construct a scenario requiring exactly that deal).
- for modular domains, convergence is guaranteed for negotiations involving 1-deals only

Sandholm. *Contract types for satisficing task allocation*. IEEE Symposium-1998.

Endriss et al.. *Negotiating socially optimal allocation of resources*. JAIR-2006.

Contract-Based Negotiation



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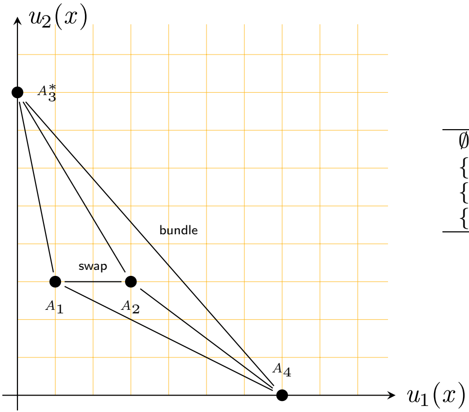
Contract-Based Negotiation



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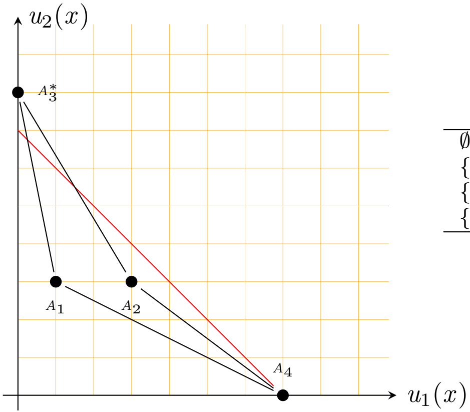
Contract-Based Negotiation



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