Learning Ceteris Paribus Preferences

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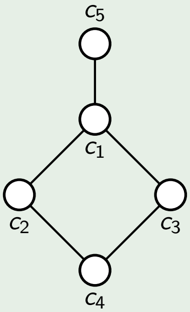
Preference context

adapted from (Brafman and Domshlak 2009)

Cars

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Preferences



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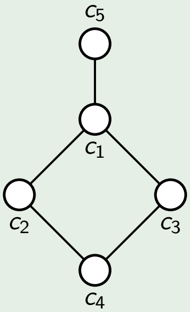

Question

You are buying a red car with bright interior. Will it be a minivan or an SUV?

From preferences over objects to preferences over descriptions

From data, derive statements like

I prefer a white car to a red car.

From preferences over objects to preferences over descriptions

From data, derive statements like

I prefer a white car to a red car.

...and back

Use derived statements to predict preferences over new objects.

From preferences over objects to preferences over descriptions

From data, derive statements like

I prefer a white car to a red car.

What exactly does this mean?

- every white car to every red car?
- most white cars to most red cars?

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Use derived statements to predict preferences over new objects.

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From data, derive statements like

I prefer a white car to a red car.

What exactly does this mean?

- every white car to every red car?
- most white cars to most red cars?

Ceteris paribus semantics

- every white car to every red car that is otherwise similar

...and back

Use derived statements to predict preferences over new objects.

Lifting preferences to propositions

in modal preference logics (van Benthem et al. 2009)

Based on a preference relation over possible worlds:

General approach

ψ is preferred to ϕ

$\Updownarrow$

worlds satisfying ψ are preferred to worlds satisfying ϕ

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One approach to ceteris paribus semantics

ψ is preferred to ϕ, Γ being equal

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every world satisfying ψ is preferred to every world satisfying ϕ that satisfies the same formulas from Γ

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One approach to ceteris paribus semantics

ψ is preferred to ϕ, Γ being equal

$\Updownarrow$

every world satisfying ψ is preferred to every world satisfying ϕ
that satisfies the same formulas from Γ

Our approach is similar, but:

- ϕ and ψ are atomic conjunctions
- Γ is a set of atomic formulas

We use formal concept analysis as a formal framework.

Formal Concept Analysis

Formal context $\mathbb{K} = (G, M, I)$

- a set of objects G
- a set of attributes M
- objects are described with attributes: the binary relation $I \subseteq G \times M$

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Derivation operators

For $A \subseteq G$:
$A' = \{m \in M \mid \forall g \in A(gIm)\}$

For $B \subseteq M$:
$B' = \{g \in G \mid \forall m \in B(gIm)\}$

Formal Concept Analysis

Formal context $\mathbb{K} = (G, M, I)$

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c_4		×	×			×
c_5	×		×			×

$\{\text{SUV}\}' = \{c_2, c_4\}$
$\{c_2, c_4\}' = \{\text{SUV}, \text{dark}\}$

Derivation operators

For $A \subseteq G$:
$A' = \{m \in M \mid \forall g \in A (gIm)\}$

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Implications

Formal context $\mathbb{K} = (G, M, I)$

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Implication

Implication $A \to B$ holds in the context (G, M, I) if $A' \subseteq B'$.

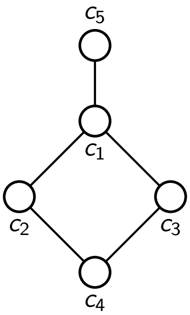
Attribute implications for cars

$$
\begin{aligned}
\text{bright} &\to \text{minivan, white} \\
\text{SUV} &\to \text{dark} \\
\text{red} &\to \text{dark} \\
\text{minivan, SUV} &\to M \\
\text{red, white} &\to M \\
\text{bright, dark} &\to M
\end{aligned}
$$

A set of implications $\Longleftrightarrow$ a Horn formula

Preference context

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×



Preference context $\mathbb{P} = (G, M, I, \leq)$

- (G, M, I) is a formal context.
- Preference relation $\leq$ is a preorder on G.

Ceteris paribus preferences

Ceteris paribus preferences in preference logics

ψ is preferred to ϕ ceteris paribus with respect to a set Γ of propositions if, for every two possible worlds w_1 and w_2 such that

- $w_1 \models \phi$,
- $w_2 \models \psi$,
- $\forall \gamma \in \Gamma (w_1 \models \gamma \iff w_2 \models \gamma)$,

we have

$$w_1 \leq w_2.$$

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we have

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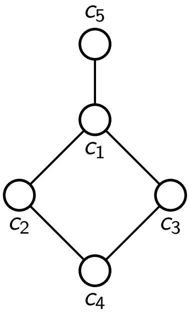
Ceteris paribus preferences in FCA $\mathbb{P} \models A \preccurlyeq_C B$

$B \subseteq M$ is preferred to $A \subseteq M$ ceteris paribus with respect to $C \subseteq M$ in $\mathbb{P} = (G, M, I, \leq)$ if

$$\forall g \in A' \forall h \in B' (\{g\}' \cap C = \{h\}' \cap C \quad \Rightarrow \quad g \leq h).$$

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×



I prefer minivans to SUVs

SUV $\preccurlyeq_{\emptyset}$minivan

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
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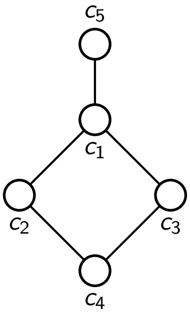
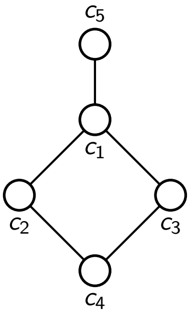


I prefer minivans to SUVs

SUV $\not\succ_{\emptyset}$ minivan

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×



I prefer minivans to SUVs — SUV $\not\preccurlyeq_{\emptyset}$ minivan

. . . with the same interior color. — SUV $\preccurlyeq_{\{\text{bright,dark}\}}$ minivan

Semantics based on preference contexts

$\mathbb{P} \models \Pi$ (Π is a set of preferences)

Π is sound for $\mathbb{P}$ $\iff$ $\forall \pi \in \Pi (\mathbb{P} \models \pi)$

$\Pi \models \pi$

π is a semantic consequence of Π if, for all $\mathbb{P}$,

$$\mathbb{P} \models \Pi \implies \mathbb{P} \models \pi.$$

Completeness

Π is complete for $\mathbb{P}$ if, for all π,

$$\mathbb{P} \models \pi \implies \Pi \models \pi.$$

Ceteris paribus preferences as implications

Ceteris paribus translation of $\mathbb{P}$

$$\mathbb{K}_{\sim}^{\mathbb{P}} = (G \times G, (M \times \{1,2,3\}) \cup \{\leq\}, I_{\sim})$$

$$\begin{aligned}
(g_1, g_2) I_{\sim} (m, 1) &\iff g_1 I m, \\
(g_1, g_2) I_{\sim} (m, 2) &\iff g_2 I m, \\
(g_1, g_2) I_{\sim} (m, 3) &\iff \{g_1\}' \cap \{m\} = \{g_2\}' \cap \{m\}, \\
(g_1, g_2) I_{\sim} \leq &\iff g_1 \leq g_2.
\end{aligned}$$

Ceteris paribus translation for cars

	m_1	s_1	r_1	...	m_2	s_2	r_2	...	m_3	s_3	r_3	...	$\leq$
...													
c_1, c_4	×					×	×						
c_1, c_5	×				×		×		×	×			×
...													

Ceteris paribus preferences as implications

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
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c_3	×			×	×	
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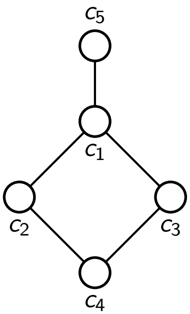

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Ceteris paribus preferences as implications

Translation of ceteris paribus preferences

A ceteris paribus preference $A \preccurlyeq_C B$ is valid in a preference context $\mathbb{P} = (G, M, I, \leq)$ if and only if the implication

$$(A \times \{1\}) \cup (B \times \{2\}) \cup (C \times \{3\}) \to \{\leq\} \tag{1}$$

is valid in $\mathbb{K}^{\mathbb{P}}_{\sim}$.

Example

$$\text{SUV} \preccurlyeq_{\{\text{bright},\text{dark}\}} \text{minivan}$$

$$\Updownarrow$$

$$\{\text{SUV}_1, \text{minivan}_2, \text{bright}_3, \text{dark}_3\} \to \{\leq\}$$

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Proposition

The set

$$\{A \preccurlyeq_C B \mid (A \times \{1\}) \cup (B \times \{2\}) \cup (C \times \{3\}) \text{ is minimal} \quad \text{w.r.t. } \mathbb{K}^{\mathbb{P}}_{\sim} \models (2)\}$$

is sound and complete for the preference context $\mathbb{P}$.

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$$\text{w.r.t. } \mathbb{K}^{\mathbb{P}}_{\sim} \models (2)\}$$

is sound and complete for the preference context $\mathbb{P}$.

Unfortunately, this set is also quite redundant.

Canonical form

Many ways to write the same preference

$$a \preccurlyeq_{cd} bd \quad \equiv \quad ad \preccurlyeq_{c} bd \quad \equiv \quad ad \preccurlyeq_{cd} b$$

$$\equiv$$

$$ad \preccurlyeq_{cd} bd$$

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$$|||$$

$$ad \preccurlyeq_{cd} bd$$

In the translated context $\mathbb{K}^{\mathbb{P}}_{\sim}$, we have

$$\mathbb{K}^{\mathbb{P}}_{\sim} \models \{d_i, d_j\} \to \{d_k\} \qquad \text{for } i \neq j \neq k \in \{1, 2, 3\}.$$

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Canonical-form preferences

$A \preccurlyeq_C B$ is in canonical form if $A \cap C = A \cap B = B \cap C$.

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Canonical-form preferences

$A \preccurlyeq_C B$ is in canonical form if $A \cap C = A \cap B = B \cap C$.

The canonical form of $A \preccurlyeq_C B$

$$can(A \preccurlyeq_C B) = A \cup (B \cap C) \preccurlyeq_{C \cup (A \cap B)} B \cup (A \cap C).$$

Ceteris paribus preferences as implications

Translation of ceteris paribus preferences

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is sound and complete for the preference context $\mathbb{P}$.

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A ceteris paribus preference $A \preccurlyeq_C B$ is valid in a preference context $\mathbb{P} = (G, M, I, \leq)$ if and only if the implication

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$$\text{w.r.t. } \mathbb{K}^{\mathbb{P}}_{\sim} \models (2)\} \text{ and } A \cap B = A \cap C = B \cap C\}$$

is sound and complete for the preference context $\mathbb{P}$,

but still redundant.

Redundancy

Example

$$\{a \preccurlyeq_c b, \quad a \preccurlyeq_d b, \quad a \preccurlyeq_c d, \quad d \preccurlyeq_c b\}$$

is redundant because

$$\{a \preccurlyeq_d b, \quad a \preccurlyeq_c d, \quad d \preccurlyeq_c b\} \qquad \models \qquad a \preccurlyeq_c b.$$

Redundancy

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In the translated context $\mathbb{K}^{\mathbb{P}}_{\sim}$, we have

$$\mathbb{K}^{\mathbb{P}}_{\sim} \models d_1 \vee d_2 \vee d_3.$$

Inference

Example

If I prefer every minivan to every SUV of the same color,

then I prefer every expensive minivan to every cheap SUV of the same color and brand.

Inference

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"Easy" inference

$$\frac{A \preccurlyeq_C B}{A \cup D \preccurlyeq_{C \cup F} B \cup E}$$

Inference

"Easy" inference

Let Π be a set of ceteris paribus preferences over M. Denote

$$\Pi^\bullet = \{D \preccurlyeq_F E \mid \exists A \preccurlyeq_C B \in \Pi (A \subseteq D, B \subseteq E, C \subseteq F)\}$$

$$\Pi^\circ = \{D \preccurlyeq_F E \mid D \preccurlyeq_F E \in \Pi^\bullet \text{ is in canonical form and } D \cup E \cup F = M\}.$$

$\Pi^\bullet$: preferences derivable from Π by the "easy inference" rule.

Π°: the weakest canonical-form preferences from $\Pi^\bullet$.

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$\Pi^\bullet$: preferences derivable from Π by the "easy inference" rule.

Π°: the weakest canonical-form preferences from $\Pi^\bullet$.

Proposition

For any preference $A \preccurlyeq_C B$ and preference set Π:

$$\Pi \models A \preccurlyeq_C B \iff \{A \preccurlyeq_C B\}^\circ \subseteq \Pi^\bullet.$$

CETERIS PARIBUS CONSEQUENCE($A \preccurlyeq_C B, \Pi$)

Input: A ceteris paribus preference $A \preccurlyeq_C B$ and a set Π of ceteris paribus preferences (over a universal set M).
Output: **true**, if $\Pi \models A \preccurlyeq_C B$; **false**, otherwise.

```
S := [can(A ≼_C B)]                                          {stack}
repeat
    D ≼_F E := pop(S)
    if A ⊆ D, B ⊆ E, and C ⊆ F for no A ≼_C B in Π then
        X := M \ (D ∪ E ∪ F)
        if X = ∅ then
            return false
        choose m ∈ X
        push(S, D ∪ {m} ≼_F E)
        push(S, D ≼_F E ∪ {m})
        push(S, D ≼_{F∪{m}} E)
until empty(S)
return true
```

Ceteris Paribus Consequence($A \preccurlyeq_C B, \Pi$)
in action

$$M = \{a, b, c, d, e\}.$$
$$\Pi = \{a \preccurlyeq_d b, \quad a \preccurlyeq_c d, \quad d \preccurlyeq_c b\}.$$

- Does $a \preccurlyeq_c b$ follow from Π?

Stack $\mathcal{S}$

Current preference $D \preccurlyeq_F E$

Ceteris Paribus Consequence($A \preccurlyeq_C B, \Pi$)

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$a \preccurlyeq_c b$

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Current preference $D \preccurlyeq_F E$

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Stack $\mathcal{S}$

Current preference $D \preccurlyeq_F E$

$a \preccurlyeq_c b \notin \Pi^\bullet$

CETERIS PARIBUS CONSEQUENCE($A \preccurlyeq_C B, \Pi$)

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Stack $\mathcal{S}$

$a \preccurlyeq_{cd} b$
$a \preccurlyeq_c bd$
$ad \preccurlyeq_c b$

Current preference $D \preccurlyeq_F E$

CETERIS PARIBUS CONSEQUENCE($A \preccurlyeq_C B, \Pi$)

in action

$M = \{a, b, c, d, e\}.$

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Stack $\mathcal{S}$

$a \preccurlyeq_c bd$
$ad \preccurlyeq_c b$

Current preference $D \preccurlyeq_F E$

$a \preccurlyeq_{cd} b$

Ceteris Paribus Consequence($A \preccurlyeq_C B, \Pi$)

in action

$$M = \{a, b, c, d, e\}.$$

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- Does $a \preccurlyeq_c b$ follow from Π?

Stack $\mathcal{S}$
$a \preccurlyeq_c bd$
$ad \preccurlyeq_c b$

Current preference $D \preccurlyeq_F E$	
$a \preccurlyeq_{cd} b \in \Pi^\bullet$	$\Leftarrow a \preccurlyeq_d b$

Ceteris Paribus Consequence($A \preccurlyeq_C B, \Pi$)

in action

$M = \{a, b, c, d, e\}$.

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CETERIS PARIBUS CONSEQUENCE($A \preccurlyeq_C B, \Pi$)

in action

$M = \{a, b, c, d, e\}$.

$\Pi = \{a \preccurlyeq_d b, \quad a \preccurlyeq_c d, \quad d \preccurlyeq_c b\}$.

- Does $a \preccurlyeq_c b$ follow from Π?

Stack $\mathcal{S}$
$ad \preccurlyeq_c b$

Current preference $D \preccurlyeq_F E$	
$a \preccurlyeq_c bd \in \Pi^\bullet$	$\Leftarrow a \preccurlyeq_c d$

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$$\Pi = \{a \preccurlyeq_d b, \quad a \preccurlyeq_c d, \quad d \preccurlyeq_c b\}.$$

- Does $a \preccurlyeq_c b$ follow from Π? Yes!

Stack $\mathcal{S}$

Current preference $D \preccurlyeq_F E$

Checking semantic consequence

- The algorithm is exponential in $|M|$.
- The theory implied by ceteris paribus preferences is generated by
  - Horn formulas into which we translate preferences;
  - $\neg m_i \vee \neg m_j \vee m_k$ for each $m \in M$ and $i \neq j \neq k \in \{1, 2, 3\}$;
  - $m_1 \vee m_2 \vee m_3$ for each $m \in M$.
- However, the algorithm is linear in $|\Pi|$.

Predicting preferences

Question

You are buying a red car with bright interior. Will it be a minivan or an SUV?

More generally

Given a preference context $\mathbb{P}$ and two additional objects g and h with descriptions A and B, predict which is better.

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One approach

Generate "minimal" canonical-form preferences Π valid in $\mathbb{P}$.

If Π contains $D \preccurlyeq_F E$ with

- $D \subseteq A$,
- $E \subseteq B$,
- $F \cap (A \vartriangle B) = \varnothing$,

then predict $g \leq h$.

If Π contains $E \preccurlyeq_F D$ with

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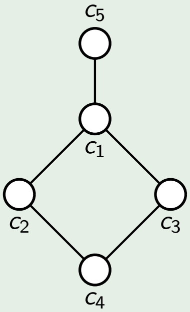
Predicting preferences

Cars

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Preferences



Which is better?

c_6: $\{\text{minivan}, \text{red}, \text{bright}\}$

c_7: $\{\text{SUV}, \text{red}, \text{bright}\}$

$\mathbb{P} \models \text{SUV} \preccurlyeq_{\text{bright,dark}} \text{minivan}$

- $c_7 \leq c_6$

Predicting preferences

Cars

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

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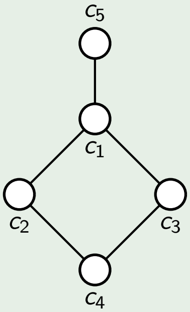

Which is better?

c_6: {minivan, red, bright}

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$\mathbb{P} \models$ minivan $\preccurlyeq_{\varnothing}$ SUV, bright

- $c_6 \leq c_7$

Predicting preferences

Cars

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
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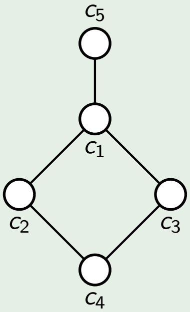

Which is better?

c_6: $\{$minivan, red, bright$\}$

c_7: $\{$SUV, red, bright$\}$

$\mathbb{P} \models \text{minivan} \preccurlyeq_{\varnothing} \text{SUV, bright}$

- $c_6 \leq c_7$ bad!

Predicting preferences

Problem (special case)

Using all "minimal" canonical-form preferences of $\mathbb{P}$, we will always predict preferential indiscernibility for a pair of objects at least one of which has a combination of attributes not recorded in $\mathbb{P}$.

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Solution

A preference $A \preccurlyeq_C B$ is supported by $\mathbb{P} = (G, M, I, \leq)$ if

- $\mathbb{P} \models A \preccurlyeq_C B$,
- $\exists g \in A' \exists h \in B' (g' \cap C = h' \cap C)$.

- We should use only preferences supported by $\mathbb{P}$ for prediction.
- Preferences supported by $\mathbb{P}$ correspond to implications $X \to_{\leq}$ of $\mathbb{K}^{\mathbb{P}}_{\sim}$ with $X' \neq \varnothing$.

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Example

minivan $\preccurlyeq_{\varnothing}$ SUV, bright $\Rightarrow$ minivan$_1$, SUV$_2$, bright$_2$ $\to_{\leq}$

don't use this preference $\{$minivan$_1$, SUV$_2$, bright$_2\}' = \varnothing$

Predicting preferences

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Given a preference context $\mathbb{P}$ and two additional objects g and h with descriptions A and B, predict which is better.

One approach

Generate minimal canonical-form preferences Π supported by $\mathbb{P}$.

If Π contains $D \preccurlyeq_F E$ with

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- This is a costly step. Can it be avoided?

If Π contains $D \preccurlyeq_F E$ with

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Given a preference context $\mathbb{P}$ and two additional objects g and h with descriptions A and B, predict which is better.

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Generate minimal canonical-form preferences Π supported by $\mathbb{P}$.

- This is a costly step. Can it be avoided? Yes.

If $\mathbb{P}$ supports $D \preccurlyeq_F E$ with
- $D \subseteq A$,
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then predict $g \leq h$.

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then predict $h \leq g$.

We can check if there is such $D \preccurlyeq_F E$ supported by $\mathbb{P}$ in time polynomial in the size of $\mathbb{P}$ using a modification of the algorithm for abduction from (Kautz et al. 1995).

Further work

- Compact representations
- Efficient algorithms
- Strict preferences
- "Ordinal ceteris paribus" conditions (via FCA scaling):

  John prefers an academic job to a job in industry (given the salary is at least as good).

- Learning preferences with queries
- Association rules instead of implications
- Preferences under incomplete knowledge
- Experimental evaluation